COPY

2

SECURITY CLASSIFICATION OF THIS PAGE

AD-A221 191

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1a. REPORT SECURITY CLASSIFICATION: UNCLASSIFIED	1b. RESTRICTIVE MARKINGS
2a. SECURITY CLASSIFICATION AUTHORITY	3. DISTRIBUTION/AVAILABILITY OF REPORT: Approved for public release; distribution unlimited.
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE	
4. PERFORMING ORGANIZATION REPORT NUMBER(S)	5. MONITORING ORGANIZATION REPORT NUMBER(S): AFOSR-TR- 90-0394
6a. NAME OF PERFORMING ORGANIZATION: Rutgers University — 6b. OFFICE SYMBOL (If applicable)	7a. NAME OF MONITORING ORGANIZATION: AFOSR/NM
6c. ADDRESS (City, State, and ZIP Code): RUTCOR, Hill Center for the Mathematical Sciences, Rutgers University, New Brunswick, NJ 08903	7b. ADDRESS (City, State, and ZIP Code): AFOSR/NM, Bldg. 410, Bolling AFB, DC 20332-6448
8a. NAME OF FUNDING/SPONSORING ORGANIZATION: AFOSR — 8b. OFFICE SYMBOL (If applicable): NM	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER: AFOSR-89-0066
8c. ADDRESS (City, State, and ZIP Code): AFOSR/NM, Bldg. 410, Bolling AFB, DC 20332-6448	10. SOURCE OF FUNDING NUMBERS

PROGRAM ELEMENT NO.	PROJECT NO.	TASK NO.	WORK UNIT ACCESSION NO.
61102F	2304	B1	

11. TITLE (Include Security Classification)

A RUTCOR Project in Discrete Applied Mathematics

12. PERSONAL AUTHOR(S)

Peter L. Hammer and Fred S. Roberts

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Year, Month, Day)	15. PAGE COUNT
Final Technical	FROM 11/1/88 TO 10/31/89	2/20/90	

16. SUPPLEMENTARY NOTATION

17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB-GROUP	

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

See Attached

DTIC
ELECTE
MAY 04 1990
S D D

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT: ☐ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT. ☐ DTIC USERS	21. ABSTRACT SECURITY CLASSIFICATION
22a. NAME OF RESPONSIBLE INDIVIDUAL: Dr. Neal D. Glassman	22b. TELEPHONE (Include Area Code): (202) 767-5026 — 22c. OFFICE SYMBOL: NM

DD Form 1473, JUN 86 Previous editions are obsolete.

SECURITY CLASSIFICATION OF THIS PAGE
UNCLASSIFIED

90 05 ... 131

Abstract

This project has been concerned with research in discrete mathematics and its applications. The work has involved theoretical developments, the development of new algorithms, and the application of discrete methods to practical problems. There have been five major areas of emphasis. The first, graph theory and its applications, has been concerned with graph coloring and stability and their applications, with special classes of graphs (such as perfect graphs, threshold graphs, competition graphs), and with the use of graphs to solve discrete optimization problems. The second area has involved discrete optimization, and has emphasized location problems, preprocessing and decomposition methods for solving discrete optimization problems, approximate algorithms for solving such problems, and applications of combinatorial optimization to nonlinear problems (global optimization). The third area of emphasis has been on combinatorial structures and their applications. Our research here has been concerned with such useful combinatorial structures as posets, combinatorial ~~designs, matroids, and 0-1~~ matrices. It has also been concerned with random discrete structures and their applications and the relations between combinatorics and other areas of mathematics. The fourth area has been the development of efficient algorithms for discrete problems. Our work on algorithms has stress~~ed~~ five themes: ~~probability and~~ algorithms, on-line methods, heuristics, approximation, and parallel and distributed computing. The fifth area of emphasis has been applications of discrete mathematics to decisionmaking. We have studied group decisionmaking and multi-person games, measurement theory and decisionmaking, and multiple conclusion logic. Among the many applications we have considered in this project are location of communication centers, clustering of data, weapons allocation, channel assignment, development of unambiguous codes, simplification of large-scale computer models, selection of routes to be served by a commercial or military carrier, reliability of distribution systems, removal of inconsistencies in database systems, and scheduling problems involving tasks, machines, or fleets.

FINAL TECHNICAL REPORT

To Air Force Office of Scientific Research

A RUTCOR Project in Discrete Applied Mathematics

Grant Number AFOSR 89-0066

Accomplishments: November 1, 1988-October 31, 1989

Peter L. Hammer
Co-Principal Investigator

Fred S. Roberts
Co-Principal Investigator

February 20, 1990

Table of Contents

A RUTCOR Project in Discrete Applied Mathematics

Grant Number AFOSR 89-0066

SUMMARY OF RESEARCH ACCOMPLISHMENTS

November 1, 1988-October 31, 1989

This summary of research accomplishments is organized into essentially the same sections and subsections as is our original proposal. Papers referred to by number are listed below in the list of publications prepared under the grant during the period November 1, 1988 to October 31, 1989.

1. Graph Theory and its Applications

Research work in the theory of graphs is closely related to a variety of applied problems. Our research has been involved with a number of graph-theoretical questions which are closely tied to applications. The applications we have considered involve primarily questions of communications and transportation and basic problems in operations research such as scheduling, maintenance, and assignment problems. The specific mathematical questions can be divided into three areas: questions dealing with coloring and stability, questions about the structure and properties of special classes of graphs, and graph-theoretical questions related to discrete optimization.

1.1. Graph Coloring and Stability

Much current work in graph theory is concerned with the related problems of finding optimal graph colorings and finding the largest stable set in a graph. Both of these problems are closely tied to practical applications and our work on them has been connected to such applications.

We have been studying T-colorings of graphs in connection with frequency assignment problems. In such problems, the vertices of a graph G represent transmitters and an edge between two vertices represents interference. We seek to assign to each vertex or transmitter x a channel $f(x)$ over which x can transmit, and for simplicity we take the channels to be positive integers. The assignment of channels is subject to the restriction that if two transmitters interfere, i.e., if the corresponding vertices are joined by an edge, then the channels assigned to these transmitters cannot be separated by a disallowed distance. To make this more precise, we fix a set T of nonnegative integers and assign channels so that if vertices x and y are joined by an edge of G, then $|f(x)-f(y)|$ is not in T. The assignment f is called a <u>T-coloring</u>. The problems we have considered were motivated by practical problems we discussed with the Air Force Frequency Management Office, the National Telecommunications and Information Administration/Washington, the National Telecommunications and Information Administration/Annapolis, the U.S. Cinceur Frequency Liason Office of NATO in Brussels, and Dr. Tim Lanfear of the NATO staff, whom we had met at NATO and invited to our Third Advanced Research Institute in Discrete Applied Mathematics at RUTCOR. We have studied the problem of finding the optimum order of a T-coloring, i.e., the minimum number of different values $f(x)$, and the problem of finding the optimum span of a T-coloring, i.e., the minimum separation between the smallest and largest $f(x)$ values.

The culmination of several years' work on the T-coloring problem was the thesis [66]. In this thesis and also in the paper [65], we have obtained new results about the parameters order and span by introducing a theory of forbidden difference graphs. In the thesis [66], we have also obtained results under special assumptions about the set T. We have developed a theory of list T-colorings, which apply to the practical problem where each user specifies a choice set of possible frequencies, from which the frequency assigned to his transmitter must be chosen. This theory generalizes the Erdos, Rubin, and Taylor [1979] theory of choice numbers. Also, building on the theory of set colorings developed by Roberts [1979b] and Opsut and Roberts [1981], we have developed a theory of set T-colorings, where each transmitter receives a set of possible frequencies, rather than a single frequency. This theory is also relevant to the various applied problems which motivated the theory of set colorings, namely, the mobile radio frequency assignment problem, the traffic phasing problem, the fleet maintenance problem, and the task assignment problem. These problems are all discussed by Opsut and Roberts [1981] or Roberts [1984].

In other work on coloring, we have continued our study, begun in Hansen and Kuplinsky [1988], of the reasons why heuristics for graph-coloring do not provide optimal colorations. This work is contained in paper [35]. Relative to a heuristic, a graph is slightly hard-to-color if some instance of the heuristic uses more colors than the chromatic number; it is hard-to-color if all instances of that algorithm do so. We have obtained small hard-to-color and slightly hard-to-color graphs by hand for many heuristics. Moreover, we have used the computer to find vertex and edge critical graphs of these kinds.

We have also been studying the problem of coloring a hypergraph. The chromatic index of a hypergraph is the smallest number of colors needed to color its edges in such a way that if two edges have a common vertex, they get a different color. We have shown in [49] that for a "nearly-disjoint" hypergraph, the chromatic index is at most $n + o(n)$.

The class of graphs called perfect was introduced by Claude Berge [1961] after he studied the interrelations of such fundamental graph parameters as the chromatic number and the stability number. Perfect graphs have become extremely important in the modern applications of graph theory. An important special class of perfect graphs, the interval graphs, have applications in numerous scheduling problems, scheduling the uses of facilities, traffic, workers, etc. They also have applications in sequencing and seriation of data, in measurement problems, in decisionmaking, in foundations of computer science, and in genetics. (A graph is called an <u>intersection graph</u> if it has an <u>intersection assignment</u>, an assignment of a set $S(x)$ to each vertex x so that distinct vertices x and y are adjacent if and only if their corresponding sets $S(x)$ and $S(y)$ have a nonempty intersection. It is an <u>interval graph</u> if there is an intersection assignment in which every $S(x)$ is a real interval.) One of the characterizations of interval graphs which goes back to Fulkerson and Gross [1965] is the following: A graph is an interval graph if and only if the rows of the (maximal) clique-vertex incidence matrix can be permuted so that the ones in each column occur consecutively. This result has led to a large literature on matrices with this consecutive ones property for columns. We have studied similar matrices with the consecutive ones property for rows. This has led to the characterization of graphs whose vertices can be linearly ordered so that every clique is a consecutive set, or so that every maximal stable set set is a consecutive set, or so that every transversal is a consecutive set. See paper [20].

1.2. Special Classes of Graphs

Many graph theory problems are extremely difficult when looked at in general, but turn out to be tractable when restricted to a special class of graphs. Hence, research in graph theory has in recent years emphasized the study of rich and interesting special classes of graphs, many of which arise from applications, and for which efficient algorithms can often be found to solve important optimization algorithms. Our work on special classes of graphs has reflected this point of view.

Among the classes of graphs we have studied are the <u>threshold graphs</u>. These graphs were defined in Chvatal and Hammer [1977] as graphs which have an assignment of nonnegative weights to the vertices so that a subset S of the vertex set is stable if and only if the sum of the weights on S is no larger than a certain threshold value. Threshold graphs have since been studied intensively. Two chapters of the fundamental book by Golumbic [1980] are devoted to threshold graphs and their natural extension, split graphs, and many references to this subject are provided by Golumbic. Among the uses of threshold graphs are applications to Guttman scaling in measurement theory (Cozzens and Leibowitz [1984]) and to synchronizing parallel processors (Henderson and Zalcstein [1977]).

A variety of authors have studied how far from being threshold a graph is. Chvatal and Hammer [1977], Erdos, Ordman and Zalcstein [1987], Cozzens and Leibowitz [1984], and Hammer and Mahadev [1985] discussed the measure of non-thresholdness called the threshold dimension, while Hammer, Ibaraki and Simeone [1981] introduced the threshold gap and Peled and Simeone [1987] introduced the threshold measure. Motivated by analogous notions in the study of Boolean functions (see paper [67]), we have introduced in paper [68] a new measure of non-thresholdness called the threshold weight of a graph. We have found the threshold weight of triangle-free graphs and have a variety of results characterizing the heavy graphs, graphs for which the threshold weight is as large as possible.

In the past year, we have studied bithreshold graphs, graphs G which are obtainable as an edge-intersection of two threshold graphs H and K such that every stable set of G is also stable in H or in K. We have found an $O(n^2)$ recognition algorithm for such graphs. We have characterized the class of bipartite bithreshold graphs as the union of five classes of graphs, and also by eleven forbidden subgraphs. (See [25].)

In paper [21], we have introduced the notion of threshold digraph. We show that the class of undirected threshold graphs, the class of Ferrers digraphs, and some other classes of graphs and digraphs, are properly "contained" in the class of threshold digraphs, and that almost all the interesting properties of threshold graphs are "valid" for threshold digraphs.

Threshold graphs have played an important role in the study of Boolean functions, to which we return in Section 2.3. A Boolean function is called a threshold function if there is a hyperplane which separates the true vectors from the false vectors. In paper [67], the notion of threshold function is generalized by considering surfaces other than hyperplanes to separate the true and false vectors. If the surface is a polynomial of degree m, we say that the threshold function has order m. In paper [69], we study the number of threshold functions of different orders.

Another class of graphs we have studied is the class of difference graphs. A graph is a difference graph if we can associate a real number a_i with each vertex i and a real number T so that $|a_i| < T$ for all i and so that if $i \neq j$, then i and j are adjacent if and only if $|a_i - a_j| \geq T$. Properties of difference graphs are described in the paper [26]. It turns out that these graphs are very similar to threshold graphs.

An important class of graphs with regard to applications is the class of competition graphs and its variants. A graph G is the competition graph of a digraph D (often assumed acyclic) if whenever x and y are vertices of D, then there is an edge between x and y in G if and only if there is a vertex a of D so that (x,a) and (y,a) are arcs of D. These graphs, introduced by Cohen [1968], arise in communications over noisy channels (cf. the confusion graphs of Shannon [1956]). They also arise in the channel assignment problem mentioned above, which is concerned with coloring a competition graph. They arise in large-scale computer models of complex systems (see for example Greenberg, Lundgren, and Maybee [1981] and Provan [1983] and Provan and Kydes [1980]). And they arise in the study of food webs in ecology. See the surveys of the literature of competition graphs by Raychaudhuri and Roberts [1985] and Lundgren [1989].

Roberts proved in [1978] that every graph is the competition graph of an acyclic digraph if sufficiently many isolated points are added to the graph. The smallest number of isolated points needed is called the competition number. An old problem has been to determine the largest competition number of a graph of n vertices. In paper [43], we solve this problem, in the process obtaining a generalization of the famous theorem of Turan about the maximum number of edges of a triangle-free graph of n vertices.

Another old problem about competition number is to settle Opsut's conjecture (Opsut [1982]). To state this, let us define $\theta(G)$ to be the smallest number of cliques which cover all the vertices of G and $N(v)$ to be the open neighborhood of vertex v. Then Opsut's conjecture says that if $\theta(N(v)) \leq 2$ for all v, the competition number of G is at most 2, with equality if and only if $\theta(N(v)) = 2$ for all v. We have proved a modified version of this conjecture in paper [58].

In recent years, there has been considerable research on variants of the notion of competition graph. We have introduced in paper [57] a variant called a p-competition graph, which arises from a digraph by taking an edge between two vertices if and only if these vertices have at least p outgoing arcs to common vertices. We have obtained results about p-competition graphs which parallel those of ordinary competition graphs. We have also introduced in paper [47] the study of the special case $p = 2$, which has led to a variety of fascinating combinatorial results.

Another variant of competition graph, introduced by Scott [1987], is the competition-common enemy graph (CCE graph), the graph obtained from an acyclic digraph by taking an edge between two vertices if and only if they have incoming arcs from a common vertex and outgoing arcs to a common vertex. The double competition number of a graph is defined for CCE graphs analogously to the competition number for ordinary competition graphs. We have investigated the double competition number of bipartite graphs, proving that it is at most 2 if one of the classes in the bipartition has at most four vertices. The result is obtained by studying 0,1-matrices which can be transformed by row and column permutations so that the pattern 1 0 1 does not appear on a diagonal. (See paper [59].)

As we pointed out in Section 1.1, of great interest in the last 25 years has been the class of perfect graphs, first introduced by

Claude Berge. These graphs arise in numerous applications, including problems involving scheduling, transportation and communications, computer systems, ecosystems, foundations of computation, genetics, and seriation in archaeology and psychology. In recent years, there has been a great deal of interest in studying special classes of perfect graphs and the relations between them. We have studied a variety of such classes. Our major piece of work on perfect graphs which was completed in the past year was the dissertation [61]. In this dissertation, we define and study some classes of perfect graphs, establish inclusion relations between them, and in most cases give polynomial algorithms for the usual problems of recognition and optimization in these classes. In several cases, we exploit the properties of Boolean functions related to the graphs under study. Among the classes of perfect graphs studied here are opposition graphs, weakly triangulated graphs, quasi-parity graphs, slim graphs, minimal non-perfectly orderable graphs, strongly perfect graphs, preperfect graphs, and completely separable graphs.

In other work on classes of perfect graphs, we have already referred in Section 1.1 to our work in paper [20] resulting from the study of the consecutive ones property which arises in the study of the special class of perfect graphs called interval graphs. Interval graphs also play a central role in the survey paper [63] which we have prepared on the applications of graph theory and combinatorics in the social and biological sciences.

Results about the class of preperfect graphs are contained in the paper [23]. (A graph is preperfect if every induced subgraph has a predominant vertex. Such a vertex x has the property that there exists a vertex y of the subgraph so that every maximum clique of the subgraph containing y also contains x or every maximum stable set of the subgraph containing x also contains y.) We show that such graphs are perfect and several well-known classes of perfect graphs are contained in the class of preperfect graphs.

An important class of perfect graphs introduced by Chvatal [1984] is the class of perfectly orderable graphs, graphs which have an order so that for each induced ordered subgraph, the greedy algorithm produces an optimal coloring. In paper [46] we establish a property of minimal non-perfectly orderable graphs, and use this property to generate a class of perfectly orderable graphs which contains the class of graphs called brittle.

Meyniel [1976] proved that a graph is perfect if every odd cycle with at least five vertices has at least two chords. Graphs with this property are called Meyniel graphs. A slim graph is any graph obtained from a Meyniel graph by removing all the edges of a given induced subgraph. We study the properties of slim graphs in paper [45], partially solving some problems of Alain Hertz.

Meyniel graphs are also studied in paper [5]. Here, we prove a conjecture of Meyniel's that every Meyniel graph has a certain kind of orientation.

Golumbic and Goss [1978] introduced chordal bipartite graphs, i.e., bipartite graphs where every cycle of length at least six has a chord. We give a characterization of these graphs in paper [24].

1.3 Graphs and Discrete Optimization

A rather large effort in our project has been devoted to discrete optimization, and we discuss this in detail in Section 2. However, we point out here that sometimes important classes of graphs are related to problems in discrete optimization.

A matching in a graph is a collection of edges or complete 2-vertex subgraphs which have no common endpoints. Maximal matchings on graphs and weighted graphs arise in a wide variety of applications in optimization problems which include job assignments, storage of computer programs, real estate transactions, etc. They had a classic application to pilot assignments for the Royal Air Force during World War II. In the past year, we have revised our paper [18] in which we study a generalization of matching called an odd chain packing, a collection of edge-disjoint chains of odd length such that all endpoints of these chains are distinct. We extend the augmenting chain theorem of matchings to odd chain packings and find an analogue of matching matroids. We show that we may restrict ourselves to packings by chains of lengths one or two and obtain a min-max result for such packings for the special case of trees.

A dual problem to the problem of matching is the problem of packing and covering. In papers [50, 51], we have obtained various extensions of the surprising results of Frankl and Rodl [1985] and of Pippenger and Spencer [1989] on packings and coverings.

An important concept in discrete optimization problems is a maximum tree or forest. We have studied $t(G)$, the maximum cardinality of an induced forest of a graph of n vertices. In paper [70], we have obtained a sharp lower bound on $t(G)$ for connected simple cubic graphs without triangles. Using this result, we show that Ewald Speckenmayer's conjecture that $t(G) \geq 2n/3$ for all biconnected cubic graphs G with girth 4 is true, except for two particular graphs, which we describe.

In earlier years, we began the study of the question: What conclusions about combinatorial optimization are meaningful in the precise sense to be defined in Section 5.2 below. We have extended our theory of the meaningfulness of conclusions in combinatorial optimization by studying the meaningfulness of conclusions about approximation algorithms for combinatorial optimization problems and by studying error evaluation functions or performance measures. The results are written up in paper [64] and are described in more detail in Section 5.2.

Graph-theoretical ideas arise in our work in paper [17] on updating the "basic algorithm" for pseudo-Boolean programming. This work is described in Section 2.2. We show in paper [17] that a modified version of the basic algorithm has linear-time complexity when applied to functions associated in a natural way with graphs of bounded tree-width. This class of graphs has recently received a lot of attention, due in particular to their central role in the work of Robertson and Seymour on graph minors (see e.g., Robertson and Seymour [1986]) and of Arnborg, Proskurowski, and others on table-based methods for various NP-hard problems.

Graph-theoretical ideas arise in our paper [9], which describes a new heuristic for quadratic 0-1 minimization. A graph-theoretic representation of the procedure leads to a straightforward implementation of it. The procedure is discussed further in Section 2.2.

Graph-theoretic ideas play an important role in our paper [8], in which we provide a graph-theoretic interpretation of the roof duality concept of Hammer, Hansen, and Simeone [1984]. This interpetation allows us to make considerable improvements in the roof duality bound for the optimum value of an unconstrained quadratic 0-1 optimization problem. We discuss this work in more detail in Section 2.3.

Further, using graph-theoretical techniques, we have studied the satisfiability problem for Horn clauses. Our results also have applications to expert systems, and are described in Sections 5.3. (See paper [15].)

2. Discrete Optimization

Discrete optimization problems arise in a large variety of vitally important practical scheduling, allocation, planning, and decisionmaking problems. Such practical problems have been one of the reasons that discrete optimization has become one of the most rapidly developing fields of mathematical programming. Another reason is that more and more mathematical fields (e.g., group theory, number theory, Boolean algebra, graph theory, and polyhedral combinatorics) are becoming involved in the study of such optimization problems. Efforts in the area of discrete optimization have been one of the central thrusts of our research.

2.1. Location Problems

Location problems arise whenever a large set of potential sites for placing certain units is available and a selection must be made of the sites to be utilized. Such problems arise naturally in situations like placing warehouses, satellites, communication centers, military units, or emergency services. See Hansen, et al [1987] for a recent survey. We have studied a variety of location problems.

In paper [27], revised during the past year, we have studied the location problem where we are given a spatial system of "clients'" demand functions. We propose solution methods to determine the price(s), the number, the locations, the sizes, and the market areas of the plants supplying the clients in order to maximize profit. Three alternative spatial price policies are considered: uniform mill pricing, in which the same price is charged to clients at the plant door; uniform delivered pricing, in which clients pay the same delivered price irrespective of their locations; and spatial discriminatory pricing, which is such that the firm sets client-specific prices based on their locations.

Extending these results and using also techniques of global optimization, we have studied location for profit maximization with a uniform delivered price per zone policy. See paper [37].

In paper [40], we present two polynomial algorithms for determining a Lorenz point, i.e., for maximizing an equity criterion for location, on trees and on general networks, respectively. The former algorithm has a complexity of $O(n^2 \log n)$, where n is the number of vertices of the tree, which is smaller than that of the algorithm of Maimon [1986,1988] by a factor of n. The latter algorithm is, to the best of our knowledge, the first one for that problem. It has a complexity of $O(mn^2 \log n)$, where m and n denote the number of edges and vertices of the network.

Another criterion of equity, the variance of the distribution of distances, is studied in paper [41]. Moreover, techniques used in this paper have also allowed us to find efficient points on a network for the two criteria: sum of distances to all vertices and length of the shortest path tree. Such a problem arises when investment costs in a local distribution system (for example electricity or sewage) are proportional to its length and usage and maintenance costs are proportional to distances between a central facility and the users. These results are contained in papers [41] and [42].

2.2 Preprocessing and Decomposition

Discrete optimization problems arise frequently in an unmanageable form. There are a variety of reasons for this: huge numbers of redundant variables are present in the original formulation, the coefficients in the constraints are disproportionately large, the problem involves unnecessary nonlinearities, etc. A long series of studies aimed at finding proper formulations of discrete optimization problems is contained in the papers by Bradley, Hammer, and Wolsey [1974], Dembo and Hammer [1980], Hansen and Hammer [1981], Hansen, Hammer and Simeone [1982], and Butz, Hammer, and Hausmann [1982]. One approach is to transform a given problem after some manipulation into a more structured one or a small number of more structured problems, for which good solution methods exist; see for example Granot and Hammer [1974] and Hammer, Johnson, and Peled [1974] for methods of transforming 0,1-programming problems to set covering problems and Hammer, Hansen, and Simeone [1984] for transforming quadratic 0,1-optimization problems to vertex packing problems, to give examples of what we have in mind. Our research effort has given considerable emphasis to such preprocessing and decomposition of discrete optimization problems. In particular, we have been concerned over the past year with decomposition algorithms which eliminate one variable at a time, and applying such algorithms to Boolean and pseudo-Boolean functions.

A _pseudo-Boolean function_ is a real-valued function on $\{0,1\}^n$, and a _Boolean function_ is a 0-1-valued pseudo-Boolean function. Such functions have a wide variety of practical applications. As part of our study of discrete optimization problems, we have been investigating the formulation of such problems using pseudo-Boolean functions. Many times, Boolean and pseudo-Boolean methods allow the considerable simplification of combinatorial optimization problems expressed by using Boolean and pseudo-Boolean functions. This is true for example when the Boolean function can be expressed as a threshold function. Papers [67] and [69] deal with threshold functions. We have already discussed this work in Section 1.3.

The "Basic Algorithm" for pseudo-Boolean programming was originally proposed by Hammer, Rosenberg, and Rudeanu [1963a,b] and in a more streamlined fashion by Hammer and Rudeanu [1968]. The algorithm obtains the maximum by recursively eliminating variables. With the advent of new methods, the basic algorithm has progressively fallen into oblivion during the last decade. In paper [17], we have reconsidered this algorithm and shown that it is linear for a particular class of pseudo-Boolean functions associated in a natural way with graphs of bounded tree-width. As we pointed out in Section 1.3, this class of graphs has recently received a lot of attention, due in particular to their central role in the work of Robertson and Seymour on graph minors (see, e.g., Robertson and Seymour [1986]) and of Arnborg, Proskurowski, and others on table-based reduction methods for various NP-hard problems. In our paper, we propose a new approach to the elimination of a variable, based on a branch-and-bound scheme, which enables us to cut short several steps of the basic algorithm.

In paper [2], we again consider the problem of finding local or global optima of pseudo-Boolean functions by generating an improving sequence of points. Hammer, Simeone, Liebling, and de Werra [1988] introduced a classification hierarchy for pseudo-Boolean functions and derived a number of results concerning the properties of increasing (decreasing) paths, corresponding to locally increasing (decreasing) computational algorithms for the various classes. They emphasize the problem of finding local maxima so that there is no better point within Hamming distance 1. We show that many of their results can be generalized to find local maxima so that there is no better point within Hamming distance k.

In paper [9], we study the unconstrained 0-1 minimization of a quadratic pseudo-Boolean function. We present a "DDT" heuristic for solving this problem which starts with a "devour" stage by setting the largest coefficient to 0. This is followed with a "digest" stage, in which all the logical consequences of the devour stage are derived. We finish with a "tidy up" stage in which the logical consequences derived in the previous stage are used to restate the problem in a simpler form. The properties of this heuristic are studied and computational experience is reported.

2.3. Approximation

A major theme in discrete mathematics in recent years has been to find methods for approximating solutions to problems and to find exact solutions by successive approximations. The approximation problem has been a main focus of our efforts.

We have examined how bad the gap can be between the linear and integer programming solutions to problems with 0-1 coefficients and constraints. We have done so for a special class of problems by comparing matchings with fractional matchings. We have solved the fractional version of the celebrated Erdos-Faber-Lovasz conjecture. Specifically, we have proved that if H is a nearly disjoint hypergraph on n vertices (i.e., a hypergraph with any two sets intersecting in at most one point) and $\mathcal{M}$ is the set of matchings of H, then there is a function w from $\mathcal{M}$ into the positive reals so that for all edges A in H, $\sum_{A \in M} w(M) \geq 1$ and $\sum_{M \in \mathcal{M}} w(M) \leq n$. In related work, we have shown that for a k-uniform hypergraph with weights on the edges, the maximum weight fractional matching differs from the maximum weight matching by a factor of at most $k + 1 + k^{-1}$ (and this is best possible). The unweighted case was conjectured by Lovasz and proved by Furedi some years ago. We have generalized these results to the case of non-uniform, unweighted hypergraphs. See paper [55].

One approach to approximation is to associate with a given problem a "relaxation" of it, i.e., an easy problem the solution of which provides information about the solution of the original problem. Hammer, Hansen, and Simeone [1984] associated a linear program to a discrete optimization problem and showed that it provides a bound to the original one and fixes the optimal values of some of the variables. This technique is called roof duality. We have been building on this fundamental notion of roof duality to study quadratic 0-1 optimization problems. Quadratic 0-1 optimization problems (and their generalization to polynomial 0-1 optimization problems) are among the most important combinatorial problems. The most prominent applications of such problems are in "selection" decisions. These include the selection of R&D projects, the selection of petroleum leases upon which to bid, the selection of items to be included in any volume-limited or weight-limited space (the "knapsack" problem), and the selection of routes to be served by a commercial or military carrier. Any situation in which some items need to be selected from a large group of items, and for which an additive utility can be assigned, either individually or jointly, to the items selected, can be modelled as a quadratic or more general polynomial 0-1 optimization problem.

In paper [8], we provide a graph-theoretic interpretation of roof duality. We obtain an $O(n^3)$ max-flow algorithm to compute the roof dual of a quadratic pseudo-Boolean function in n variables. We also obtain a decomposition theorem for quadratic pseudo-Boolean functions, improving the persistency result of Hammer, Hansen, and Simeone. Based on this decomposition and the iterated application of roof duality we significantly improve the roof duality bound. Computational experiments on problems up to 200 variables are also presented.

In paper [14], we describe two polynomial-time algorithms which are equivalent to the roof-duality method of Hammer, Hansen, and Simeone. We then describe a third algorithm which improves on the bound computed by the first two whenever it is different from the minimum of the function. The second algorithm can be interpreted as a sequence of applications of a simple pseudo-Boolean operation called exchange. Similarly, the third algorithm is based on an operation called pseudo-Boolean consensus, which generalizes exchange.

In related work, in paper [7], we generalize three different approaches to obtain upper bounds for the maximum of a quadratic pseudo-Boolean function f over $\{0,1\}^n$. The original approaches (complementation, majorization, and linearization) were introduced by Hammer, Hansen, and Simeone, who showed that they yield the same bound. Our generalization yields three upper bounds which may be different and we explore their relationships.

We have already referred in Section 1.3 to our work on the meaningfulness of conclusions about combinatorial optimization. In particular, we have been examining in paper [64] the meaningfulness of conclusions about approximation algorithms for combinatorial optimization problems and the meaningfulness of statements involving error evaluation functions or performance measures. We discuss this further in Section 5.2.

2.4. Applications of Combinatorial Optimization to Nonlinear Problems

In operations research, one makes the distinction between algorithms designed to find a local optimum and algorithms designed to find the global optimum. The vast majority of nonlinear programming algorithms belong to the first category, but increasing attention is being devoted to the latter one. Many practical problems in the engineering literature can be looked at as constrained global optimization problems. We have found that many of the ideas underlying algorithms for combinatorial optimization could be transposed to the field of global optimization.

Our major work on the applications of combinatorial optimization to nonlinear problems has been the thesis [60], which studies several areas of global optimization. The global optimization problem is to solve the following problem: minimize (or maximize) $f(x)$ such that $x \in S$, where f is a real-valued function of n variables and S is a subset of $\mathbb{R}^n$. In general, this problem is NP-hard and very difficult to solve in practice. Much of the thesis is concerned with the case where $n = 1$ (the <u>univariate case</u>) and where f is a <u>Lipschitz function</u>, i.e., f is defined on an interval $[a,b]$ and for all x, y in $[a,b]$,

$$|f(x) - f(y)| \leq L|x-y|,$$

where L is a constant. An analytical expression for f may not be known; it may be given, for instance, by an oracle. Such a problem is interesting due to its simplicity, but also because it arises in many applications. For instance, it corresponds to the optimization of performance of systems, which can often be measured for given values of some parameter(s) even if the governing equations are unknown. Examples are the maximization of yield in agriculture, a function of the amount of fertilizers used, and the optimal tuning of electronic apparatus. Moreover, many multivariate global optimization problems become easy to solve once the values of one or of a few variables are fixed. They can thus be viewed as implicitly defined global optimization problems in these variables only. Examples are the location of plants to maximize profit subject to uniform mill or delivered pricing policies (see paper [27], which was discussed in Section 2.2) and determination of the optimal departure time for a commuter in a congested network. In the thesis, we develop a variety of algorithms for dealing with this global optimization problem. We also study more general problems where the function f being optimized is a univariate function which can have derivatives up to order 6. And we present an analytical method for dealing with global optimization problems which are multivariate and have constraints.

The global optimization of univariate Lipschitz functions is also studied in two papers, [30] and [31]. We consider such problems as finding the globally optimal value of f; finding a globally ϵ-optimal value of f and a corresponding point; localizing all globally optimal points; finding a set of disjoint subintervals of small length whose union contains all globally optimal points; and finding a set of disjoint subintervals containing only points with a globally ϵ-optimal value and whose union contains all globally optimal points. We summarize and discuss algorithms from the literature, presenting them in a simplified and uniform way, in a high-level computer language, and we introduce new algorithms. Extensive computational comparison of algorithms is also presented.

Building on papers by Hansen, Jaumard, and Lu [1989a,b,c], which were supported by an earlier AFOSR-grant, we have studied in paper [29] the extent to which global optimization problems can be solved using analytical methods. To this end, we propose a series of tests, similar to those of combinatorial optimization, organized in a branch-and-bound framework. The first complete resolution of two difficult test problems illustrates the efficiency of the resulting algorithm. Computational experience with the program BAGOP, which uses the computer algebra system MACSYMA, is reported on. One hundred test problems from the compendiums of Hock and Schittkowski [1981] and others are solved.

The analytical methods developed in paper [29] are applied to bilevel linear programming in the paper [34]. Bilevel programs arise in situations where multiple decisionmakers with divergent objectives intervene in decisions to be made. We discuss this work in some detail in Section 5.1.

At the end of the 1950's, Fortet [1959,1960] stressed the usefulness of Boolean methods in the formulation and solution of operations research problems involving qualitative decision variables. Such problems may be expressed, in the most general case, as nonlinear programs in 0-1 variables with nonlinear constraints. Several approaches to their solution have been extensively studied during the last thirty years. The four main ones are linearization, algebraic, enumerative, and cutting-plane methods. In the paper [32], we survey more recent developments. We then compare the efficiency of various approaches through extensive computational experiments.

3. Combinatorial Structures and their Applications

Combinatorial structures such as matroids, graphs, block designs, and partially ordered sets have a wide variety of applications in practical problems. Our work on such combinatorial structures has emphasized graphs (see Section 1). We have, however, also found block designs, posets, matroids, 0,1 matrices, clutters, and hypergraphs useful in our work. An increasingly important theme in discrete mathematical research is to investigate random structures of various kinds. We have studied a variety of questions involving random structures and probabilistic approaches to combinatorial problems. We have also found combinatorial methods useful in studying problems which are usually looked at from non-combinatorial points of view.

3.1. Useful Combinatorial Structures

As mentioned above, combinatorial structures of all kinds are important in a wide variety of problems. In this section, we describe our work on combinatorial designs, matroids, clutters, 0,1 matrices, posets, hypergraphs, and other combinatorial structures.

One important combinatorial structure is a poset. Posets are among the fundamental objects of discrete mathematics. They have applications to the theory of computation, optimization, game theory, preference and decisionmaking, etc. We have studied a variety of problems concerning posets. For example, suppose that P is a poset and $p(x<y)$ gives the fraction of linear extensions of P in which $x < y$. We have shown in paper [53] that any poset P which is not a chain contains x and y with

$$\frac{1}{2e} < p(x<y) < 1 - \frac{1}{2e}.$$

The proof is very simple and is based on the Brunn-Minkowski Theorem. The result is not as good as the result

$$3/11 < p(x<y) < 8/11$$

given some time ago by Kahn and Saks [1984]. However, the argument here is far simpler and in particular is the easiest way known of proving the existence of some positive constant δ for which the above statement holds with

$$\delta < p(x<y) < 1-\delta.$$

(Such a result is what is needed in computer science applications.)

In paper [22], we have studied large antichains in posets. We have discovered the surprising fact that (for large enough n), there exist antichains in $2^{[n]}$ having size at least $c2^n$ with c approximately e^{-2}. The proof is nonconstructive. Furedi and Kahn had earlier given a constructive lower bound of about $n^{-1/6}\,2^n$, disproving the upper bound of $O(n^{-1/2}\,2^n)$ conjectured by Engle. An upper bound of the form $(1-\epsilon)2^n$, ϵ a postive constant, has recently been obtained by Kostochka.

Order relations have also been studied in the paper [3]. Here we have studied the minimum reversing set in a digraph representing a preference order. This is a minimum set of arcs whose reversal makes the preference order into a transitive tournament. This concept also arises in the engineering literature, where, it is called a feedback arc set. We show that every acyclic digraph D is the minimum reversing set of some preference order which defines a tournament and study the reversing number r(D) of D, the smallest number of vertices in such a tournament.

Another important combinatorial structure is a clutter. A clutter is an ordered pair (V(H),E(H)), where V(H) is a finite set and E(H) is a set of subsets of V(H) such that no set in E(H) is contained in another. There is a natural correspondence between clutters and monotone Boolean functions. We have already discussed the significance of Boolean functions in Section 2.2. A clutter is defined to be k-monotone, completely monotone, or threshold if the corresponding Boolean function is k-monotone, completely monotone, or threshold. These various types of Boolean functions were introduced in the early 1960's because of the study of threshold Boolean functions (see Sections 1.2 and 2.2). A characterization of k-monotone clutters in terms of excluded minors is presented in paper [19]. This result is used to derive a characterization of 2-monotone matroids, and of 3-monotone matroids (which turn out to be all the threshold matroids). (Matroids, of course, are another interesting combinatorial structure, which have proven to be of crucial importance in the foundations of computational complexity.)

Still another interesting combinatorial structure is a hypergraph. We have already described in Section 1.1 our results in paper [49] on coloring of hypergraphs.

Matrices of 0's and 1's have played an important role in discrete mathematics. We have already discussed in Section 1.1 the 0,1 matrix called the (maximal) clique-vertex incidence matrix, which has played such a central role in the characterization of interval graphs. (See paper [20].) We have discussed in Section 1.2 our reduction (in paper [59]) of the problem of determining the double competition number of a bipartite graph to the question of whether every 0,1 matrix can, by row and column permutations, be reduced to a 0,1 matrix with the pattern 1 0 1 not appearing on a diagonal.

Matrices of +1's and -1's are also of interest. Random ± 1 matrices are studied in paper [52], where we study the probability that such a matrix is singular.

Matrices are also important in the theory of qualitative stability. Here, we study a system of homogeneous linear differential equations with constant coefficients defined by a matrix, and we study the stability of a solution. If the stability of the solution depends only on the sign pattern of the matrix, we say that the system and the matrix are qualitatively stable. The notion of qualitative stability has been studied at great length in part because of its many implications, for economics, ecology, etc.

The problem of characterizing qualitatively stable matrices has been solved using signed digraphs. The literature on this subject, and its various applications, has been surveyed in the paper [63].

Combinatorial designs have played a role in our work as well. Combinatorial designs arise from practical problems in experimental design, and have led to theories which are very critical to the design of error-correcting codes for communicating with unmanned missiles and rockets in the atmosphere or in space. Among the most important combinatorial designs are the projective planes. In Section 3.2, we shall describe a result about the choice of a random set of lines in a projective plane, which we obtained in paper [48].

We have also used combinatorial designs to study the λ-hyperfactorization of the complete graph K_{2n}. This is a collection of 1-factors for which each pair of disjoint edges appears in precisely λ of the 1-factors. Such a λ-hyperfactorization is called trivial if it contains each 1-factor of K_{2n} with the same multiplicity and simple if each 1-factor appears at most once. Cameron [1976] and Jungnickel and Vanstone [1987] had found examples of nontrivial λ-hyperfactorizations for special values of n. In paper [10], which is a revised version of a paper prepared in an earlier year, we have shown the existence of nontrivial, simple λ-hyperfactorizations of K_{2n} for all $n \geq 5$.

We have also found uses for generating functions and Stirling numbers, important tools of combinatorics, in our research. A difference graph (not the same as the difference graph defined in Section 1.2) is a bipartite graph such that all the neighorhoods of one of the classes in the bipartition are comparable by inclusion. In paper [62], we enumerate labeled difference graphs by the size, number of isolated vertices, and number of distinct vertex-degrees of each of the classes. The results use generating functions and lead to counts expressed in terms of Stirling numbers of the second kind.

3.2. Random Discrete Structures and their Applications

An increasingly important theme in discrete mathematical research is to investigate random discrete structures of various kinds. The reason for the emphasis on random structures is in part because of their connections to probabilistic algorithms and in part because of their relevance in formulating models for applied problems. Moreover, sometimes a probabilistic approach can lead to useful results about inherently non-probabilistic problems. We have worked on a number of problems in this area, some concerning behavior of random objects, and others whose solutions seem likely to require a probabilistic approach.

Among the more interesting applications of discrete mathematical methods are availability problems for power systems. In the paper [11] we characterize the availability of a power system by a minimum set of linear inequalities involving the generation and arc capacities as well as random variables representing deficiencies and demands. We also present a new method for evaluation of the reliability of such a system. The availability analysis is based on a classical theorem of Gale [1957] and Hoffman [1960], while the reliability evaluation is based on a new probability approximation scheme due to Prekopa [1988, 1990].

The second largest eigenvalue (λ_2) of a regular graph and the second largest absolute eigenvalue (ρ) are intimately connected with two graph features of considerable interest in computer science, namely expansion and rapid mixing. Expansion, in particular, has played a central role in many recent developments in the theory of computation, and its significance is by now well documented. The importance of rapid mixing of random walks is also becoming more apparent. It is well known (and easy to see) that a random graph is (with high probability) a good expander. What is less obvious, and what we prove in paper [56], is that it also has an essentially optimal (i.e., smallest possible) λ_2 and ρ and therefore has certifiably good expansion and mixing properties.

In paper [48], we study projective planes of order $r-1$, and we study the properties of a random set of lines from such a plane. Suppose $\tau(H)$ is the size of the smallest set of points meeting all lines in the set of lines H. We show that there is a constant C such that if H is a random set of $m > Cr\log r$ lines from such a plane, then with high probability $\tau(H) = r$.

In paper [52] we study random matrices of +1's and -1's. We study the probability that such a matrix will be singular.

3.3. Relations between Combinatorics and other Areas of Mathematics

One of the themes in modern discrete mathematics and discrete optimization research is the investigation of the interface between these areas and other areas of mathematics. We have investigated some problems of algebra, topology, and numerical analysis using combinatorial methods. Also, it should go without saying that, where needed to solve combinatorial problems, we have used methods of numerical analysis, algebra, topology, and analysis.

We have been concerned with the general problem of finding efficient ways to represent all piecewise polynomial functions which are smooth of order r and are of degree at most m, over a d-dimensional triangulated region in d-space. Such functions, often called <u>splines</u> or <u>finite elements</u>, have application in computer graphics and surface modeling as well as in numerical analysis. (For background, see Billera [1988,1990].) We have been concerned with the problem of determining the dimensions of such spaces as well as computationally useful bases. In attacking this problem, we have used methods from combinatorics and commutative algebra similar to those used to study the numbers of faces of convex polytopes. We have related the study of the ring of continuous piecewise polynomials over a d-dimensional triangulation to the study of the face ring of the underlying simplicial complex and showed how this leads directly to a specification of the dimensions in question. Our results on representations of smooth piecewise polynomial functions over triangulated regions have led in particular to the conclusion that Groebner basis methods of computational commutative algebra might lead to effective means to compute dimensions and bases of spline spaces in large practical examples. See paper [4].

Other applications of combinatorial methods outside of combinatorics have already been described in Section 3.2, where we have discussed the use of combinatorial methods in the solution of problems of reliability theory. (See paper [11].)

4. Efficient Algorithms for Discrete Problems

One of the major changes in discrete mathematics in the 1970's and 1980's has been the strong emphasis on algorithms. Throughout the project we have reflected this emphasis by studying algorithms for a variety of discrete problems. The study of complexity of algorithms has become a subject in its own right, and some of our research can be classified as the study of the foundations of computational complexity. We have also tried to emphasize several themes which we see as increasingly important. These themes are probability and algorithms, on-line methods, heuristics, approximation, and parallelism. We have not done much research on any of these themes directly in the sense of having, say, a study of the "foundations of probabilistic algorithms." Rather, we have tried to bring to bear on a variety of discrete problems approaches of the kind described in this section.

4.1. Probability and Algorithms

It has long been known that many algorithms which can be bad in their worst cases are very good in an "average" case. This has led to increased interest in analysis of algorithms over random instances of problems. Here, the inputs are drawn from a known distribution and we seek algorithms with good average case behavior. Recent studies of the average case behavior of the simplex algorithm are important examples of what we have in mind. Probabilistic ideas enter into the development of efficient algorithms in another way as well. Namely, sometimes if we allow a machine to make some random choices, we obtain an algorithm -- a random algorithm -- which is very effective at solving a problem.

We have been working on a probabilistic approach to the maximum satisfiability problem. In this problem, we seek the largest possible set of logical clauses from a given set which may be simultaneously satisfied. (See Section 5.3 for more details.) This problem contains the satisfiability problem and is NP-complete even when all the clauses contain at most two literals. In paper [13], which was updated from earlier work, we have developed a probabilistic model for MSP and apply probabilistic bounds to develop Branch-and-Bound type algorithms for their solution. (It is interesting to note that the probabilistic existence proof results in a deterministic algorithm.) The probability bounds prove the existence of some solutions having reasonably good quality. (Further work on the satisfiability problem is contained in paper [15] and is described in Section 5.3.)

One of the most important problems in combinatorial optimization is the problem of finding an optimal flow through a network, a transportation network, an electrical network, a pipeline network, etc. We have been studying this problem probabilistically in paper [12]. Here we obtain results about the probability of the existence of a feasible flow in a transportation network.

Current research on properties of and algorithms for 0-1 quadratic programming often uses randomly generated test problems as surrogates for actual applied problems. The properties and algorithms often concentrate on local maxima, but until now no one has had much idea of how many local maxima we have to contend with. Some of our results in paper [1] indicate that there are many more than most people thought, when we get to dimensions of 100 or more. Moreover, as we report in that paper, we have had success in estimating the expected number of local maxima for the randomly generated problems of the type that many researchers use as test problems. This knowledge should, in many instances, affect the thinking that goes into the design of algorithms.

4.2. On-Line Methods

There is increasing emphasis in practical problems to find solution algorithms which are on-line in the sense that one is forced to make choices at the time data becomes available, rather than after having the entire problem spelled out.

To be more specific, we can say that a generic combinatorial optimization problem involves selecting the optimal configuration for a system from among a set of allowed configurations. Typically, techniques for solving such problems assume that the entire system is known in advance, before any decisions have to be made. In many cases, as we have pointed out, this assumption is unrealistic; the optimization may involve a sequence of decisions, some of which must be made before knowing all of the parameters of the system. Sequential decision procedures which operate in such an environment are called on-line algorithms. Such algorithms are relevant, for instance, when decisions must be made in response to a dynamically changing system. There are two basic approaches to such decision problems. A common approach is to formulate a probabilistic model of the future and to minimize the expected cost of the decisions. This is the starting point of the theory of stochastic optimization for which there is a vast literature. An alternative approach involves no such probabilistic assumptions. Here, we evaluate an on-line decision strategy by comparing its performance on every input to that of the optimal off-line algorithm, i.e., one that works with complete knowledge of the future. Of course, in general, we expect that the on-line strategy will perform much worse in the worst case than the off-line strategy. However, a number of cases have been identified in which there are on-line algorithms whose performance can be tightly bounded as a function of that of the optimal off-line algorithm.

We have briefly studied in the past year the problem of coloring graphs on-line. We have studied various approaches to obtaining on-line algorithms for T-colorings, as defined in Section 1.1. In thesis [66], we point out that some of the ordinary on-line coloring results of Gyarfas and Lehel [1988] can be extended to on-line T-colorings. These results show that there are on-line algorithms whose performance is, indeed, quite closely bounded by that of the optimal off-line algorithm.

Another on-line optimization problem which we have studied in paper [16] is enumeration of the new vertices of a polytope when it is cut by a hyperplane. On-line vertex enumeration is a basic step in many methods for concave programming (Falk and Hoffman [1986]) and d-c programming (Tuy [1986]), as well as for global optimization (Thach and Tuy [1987]). Instead of a simplex based method, we generate new vertices by computing intersection points of edges with the new hyperplane. Computational results show that this method is quicker than that of Dyer [1983] for off-line enumeration of vertices, an unexpected result.

4.3. Heuristics

As more and more problems are shown to be difficult, for instance by proving them to be NP-complete, there is coming to be an increasing emphasis on heuristic solutions. Heuristic algorithms are especially important in practice where there are many problems involving hundreds, thousands, even tens of thousands of variables. In such a case, we would like to elaborate a heuristic algorithm capable (in most cases) of very rapidly finding (approximate) solutions to large problems.

We have already described in Section 2.2 our work in paper [9] on the DDT heuristic for quadratic 0-1 minimization.

We have described in Section 1.1 our work in paper [35] on heuristics for graph coloring.

Many of the algorithms developed in our work on global optimization, described in Section 2.4, are heuristics. See in particular the thesis [60].

In paper [39], we describe heuristics for the 0-1 hyperbolic sum problem, which arises in the problem of determining the optimal set of queries in a database. This work is described in Section 5.3.

4.4. Approximate Algorithms

As we observed in Section 2.3, a major theme in discrete mathematical research in recent years has been to find methods for approximating solutions to problems and to find exact solutions by successive approximations. The approximation problem has been a main focus of our efforts, at least for discrete optimization problems. We see no need to elaborate on the already detailed discussion in Section 2.3, except to mention two results on approximate algorithms which are not mentioned there. One such is described in the paper [38] and elaborated on in Section 5.3. It has to do with approximate algorithms for solving the problem of finding the optimal logical form of a query in a classical database. The other is described in the paper [64] and elaborated on in Section 5.2. It has to do with the meaningfulness of conclusions about approximate algorithms and of expressions involving error evaluation functions concerning such algorithms.

4.5. Parallel and Distributed Computing

The design, analysis, and management of computing systems that consist of many processors is a central part of computer science, and its study has led to many important theoretical and mathematical problems. Such systems divide into two general (and not entirely distinct) categories, distributed systems and parallel computers. A distributed system consists of autonomous, physically separated computers linked together in a network, and the main theoretical issues center around problems of communication and synchronization of such systems. A parallel computer is usually a single machine composed of many distinct processing units which work together to perform the same kinds of tasks performed by a standard sequential machine.

Theoretical research has focused on the following fundamental question: for which algorithmic problems can the required computation time be significantly reduced by the use of multiple processors (and to what extent)? Over the past several years, the catalogue of problems with efficient parallel algorithms has expanded greatly, including several fundamental algebraic and combinatorial problems. Developing such algorithms for a specific combinatorial or algebraic problem has required exploiting its mathematical structure in order to efficiently divide the work among the processors. The new insights gained from analyzing problems from the parallel/distributed point of view have, for some problems such as the network flow problem, led directly to new efficient sequential algorithms.

The design and analysis of distributed computing networks involves many of the same fundamental ideas and problems that have been previously studied in relation to other kinds of communication networks, e.g., connectivity, reliability, routing, spanning trees, etc. In addition, understanding and tracking how information flows in the network and how the individual processors respond to this information requires the development of new logical and combinatorial models. Issues of fault tolerance and secure communication have overlapped with the area of multiparty protocols in cryptography.

Some of the same issues arise in parallel computers, where the architecture of a specific machine is such that the processors are linked in some network structure, such as a hypercube, and thus information transfer becomes an important theoretical consideration. The effective utilization of such machines requires a thorough understanding of their underlying graph-theoretic structure.

We have not made a major effort in the area of parallel and distributed computing. Rather, we have tried to be aware of the issues involved, and to take a parallel or distributed point of view when that is appropriate.

We have, however, done two specific pieces of relevant work. Paper [36] is based on the work of Bokhari [1987,1988], who has studied the assignment of the modules of a parallel program to the processors of a multiple-computer system. He has proposed algorithms to solve optimally the following problems: (1) partition chain-structured or pipelined programs over chain-connected systems; (2) partition multiple chain-structured parallel or pipelined programs over single-host multiple-satellite systems; (3) partition multiple arbitrarily structured serial programs over single-host multiple-satellite systems; (4) partition single-tree structured parallel or pipelined programs over single-host multiple identical satellite systems. In paper [36], we solve problem (1) by dynamic programming and problem (2) by sorting and using bisection search for the bottleneck value. We also note that Bokhari's algorithms for problems (3) and (4) can be improved by using recent results of Gallo, Grigoriadis, and Tarjan [1989] and by implementing Dijkstra's algorithm, which is used as a subroutine, with a heap structure. The time complexity of all algorithms is thus reduced.

In paper [54], we have studied a problem in the foundations of computational complexity which, though not explicitly a problem about parallel or distributed computing, is still in the spirit of some of the questions we have posed above. Namely, we have studied the conjecture that, stated imprecisely, says that if you want to compute even a very simple function with a certain kind of Boolean circuit (a bounded depth Boolean circuit with gates with output functions depending only on a restricted sum of inputs), the circuit must be very large. We have proven this conjecture for a special case.

5. Applications of Discrete Mathematics to Decisionmaking

Problems involving complex choices are often most naturally formulated using discrete mathematics. The tools of discrete mathematics are widely used in the literature of individual decisionmaking, group decisionmaking, game theory, and measurement and utility theory. We have made progress using discrete techniques on several problems involving decisionmaking and have also found concepts from the theory of decisionmaking to be useful in attacking other problems of discrete mathematics.

5.1. Group Decisionmaking and Multi-Person Games

Many decisions are made by groups, either acting under strict decisionmaking rules or acting independently, but governed by certain constraints. The theories of group decisionmaking and multi-person games have been developed to deal with these situations. The problems which arise in group decisionmaking and game theory are frequently of a discrete nature, and we have studied a number of them with an eye on important applications.

In many situations, multiple decisionmakers with divergent objectives intervene in the decisions to be made. The simplest such case, in which there are only two decisionmakers, has long been studied in game theory. If there is some dissymmetry between the decisionmakers, in that one of them, called the *leader*, makes his or her decisions first, anticipating the reaction of the other one, called the *follower*, and cooperation is ruled out a priori, one has a Stackelberg game. Adding joint constraints on the strategies of the leader and the follower makes the model more realistic. This leads to *bilevel programming*, a topic which has attracted much recent attention, both in the linear case and in the nonlinear one. Applications of bilevel programming have been made in many domains. These include economic development policy, agricultural economics, road network design, and cogeneration of thermal and electrical energy. Mathematical programs with optimization problems in the constraints, arising in weapons allocation problems (see Bracken and McGill [1973, 1974], Bracken, Falk, and McGill [1974]), can also be viewed as (nonlinear) bilevel programs. In the paper [34], we propose a new algorithm for bilevel linear programming. The algorithm uses elimination of variables controlled by the follower, within a branch-and-bound framework. Necessary optimality conditions expressed in terms of tightness of the constraints are used to fathom or simplify subproblems, branch, and obtain penalties similar to those used in mixed-integer programming. Computational results are also reported on in this paper.

Most location problems on networks deal with efficiency criteria such as minimizing travel cost or time to a facility for all users or for the least favored one, maximizing number of users within a given distance to a facility, and the like. Recently, attention has begun to be paid to equity criteria of the kind commonly dealt with in the literature of game theory and group decisionmaking (social welfare). The focus is then on the distribution of distances from the users to the facility and measures of its dispersion. In paper [40], we present two polynomial algorithms for determining a Lorenz point, i.e., for maximizing an equity criterion for location, on trees and on general networks, respectively. Other equity criteria are studied in papers [41] and [42]. (See more details in Section 2.1.)

In paper [63], we have surveyed recent work on social welfare functions, with some emphasis on variants of the famous Arrow axioms and on the Kemeny-Snell means and means, which have had a variety of applications in practical problems, including a recent application in locating facilities in Geneva. We have emphasized three important directions of research. One involves the calculation of social welfare functions when profiles are derived from spatial representations of both alternatives and group members. The second has been to apply combinatorial optimization techniques to problems of social welfare functions. And the third is to invoke the theory of meaningful statements from measurement theory (see Section 5.2) to choose social welfare functions which lead to meaningful comparisons.

5.2. Measurement and Decisionmaking

Mathematical theories of measurement and decisionmaking have long been important in both the physical and the social sciences, and have had a wide variety of applications in economics, policy science, and environmental science, to name a few areas. The mathematical problems which arise in the theories of measurement and decisionmaking are often of a discrete nature. For a discussion of some of these, see Roberts [1985b,1989] and Fishburn and Roberts [1989]. We have been applying some of the methods of the theories of measurement and decisionmaking to study important decisionmaking problems.

An area of considerable current interest in the theory of measurement is the theory of meaningful statements. Put briefly, a statement is called meaningful if its truth or falsity is independent of the particular versions of scales of measurement used. That is, its truth or falsity is not an artifact of the particular scales of measurement used. (See Roberts [1979a] for detailed definitions of concepts from the theory of measurement.) The theory of meaningfulness has had a wide variety of applications, including applications involving average performance measures for new technologies, importance ratings, indices of consumer confidence, psychophysics, social networks, and structural modeling for complex decisionmaking problems. See Roberts [1985a] for a survey. In earlier years, we began to look at the limitations which the requirements of meaningfulness place on conclusions from combinatorial optimization problems. In paper [64],we have continued this work, updating our earlier paper on the subject, and in particular obtaining a variety of new results about the meaningfulness of conclusions about approximation algorithms and about the meaningfulness of conclusions involving error evaluation functions or performance measures. To describe some of this work, let us consider the problem P of maximizing a function $f(x)$ for x belonging to a set A. Suppose $E(v,P)$ measures the error in obtaining a solution v in A to the problem P. This is called an error evaluation function. Following some work of Zemel [1981], we were led to consider the meaningfulness of assertions like

$$(*) \qquad E(v,P) > E(w,P).$$

We consider several different error evaluation functions E. For instance, we consider $E(v,P) = [f(z)-f(v)]/|f(z)|$, the so-called relative error. If $f(x)$ is measured on a ratio scale, then we show that (*) is meaningful. However, if $f(x)$ is measured on an interval scale, it is not. To mention another result, suppose that G is an approximation algorithm for problem P and $v_G(P)$ is the value obtained for problem P by algorithm G. A common assertion is that for all problems P of a given class, $E(V_G(P),P) \leq r$, where r is some constant. This is the assertion that the error in the solution given by the algorithm is bounded for all problems of a given class. We evaluate the meaningfulness of this statement.

Much of measurement and decisionmaking begins with clustering or partitioning of alternatives into groups. Our work on measurement has put considerable emphasis on clustering or partitioning problems. Clustering methods aim at finding within a given set of entities, subsets called clusters which are both homogeneous and well-separated. These concepts can be made precise in terms of dissimilarities between entities. The split of a cluster is the smallest dissimilarity between an entity in that cluster and one outside it. The diameter of a cluster is the largest dissimilarity between two entities in that cluster. In paper [28], we consider the problem of finding a clustering which maximizes the average split of the clusters, or equivalently the sum of splits. We provide an $O(N^2)$ algorithm for the maximum sum of splits clustering into M clusters, for all M between $N-1$ and 2, where N is the number of entities to be classified.

In some clustering applications, it is desirable for operational reasons to impose additional constraints on the clusters. The constraints most often considered are bounds on the cardinality of the clusters, though connectivity constraints when regions in the plane are clustered are also commonly studied. Cardinality or weight constraints appear naturally in many applications. These include grouping machines into cells or jobs into families in production planning, assigning files to storage devices in data management, and locating obnoxious facilities. In paper [33], we consider weight-constrained clustering. The entities to be classified have weights. We study the problems of finding the maximum split partitions with exactly M clusters and with at most M clusters, subject to the additional constraint that the sum of the weights of the entities in each cluster never exceeds a given bound. These two problems are shown to be NP-hard and reducible to a sequence of bin-packing problems. An $O(n^2)$ algorithm for the particular case $M = N$, where N is the number of entities to be classified, is presented for the second problem. We also report on computational experience.

We have also studied weight-constrainted clustering with a criterion of homogeneity, i.e., the diameter. Specifically, we find methods for obtaining the minimum diameter weight-constrained clustering. See paper [44].

5.3. Multiple Conclusion Logic

When a given signal can be interpreted as being the result of a variety of causes and a small number of tests have to be created to identify the exact cause of the signal, we have a typical instance of a multiple conclusion logic situation. Examples of such situations occur in medical decisionmaking, in searching and seeking in hazardous or nuclear or chemically toxic environments, in detecting enemy positions, in remote operations in space or underseas and so on. See Sheridan and Ferrell [1974, Part III]. We have been working on a number of approaches to such problems.

A natural approach to problems of multiple conclusion logic is based on Boolean methodology. This would devise "cheapest" tests for identifying the causes of such phenomena. Our work on Boolean methodology in general is described in detail in Section 2.2. We have begun applying Boolean methodology to the design of "inference machines" for expert systems.

Other work on multiple conclusion logic is also related to expert systems. As the size of databases and knowledge bases in expert systems grows, the occurrence of inconsistencies becomes more and more likely. It is then desirable to restore consistency by relaxing as few logical conditions as possible. In the domain of propositional calculus, this problem corresponds to the maximum satisfiability problem. To be precise, let us say that a Boolean function f is in <u>disjunctive normal form</u> if

$$f(x_1,x_2,\ldots,x_n) = \bigvee_{i=1}^{m} C_i,$$

where each C_i is the conjunction of some literals (variables or their complements). The <u>satisfiability problem</u> is to determine whether there exist $x_1, \ldots, x_n$ in $\{0,1\}$ such that $f(x_1,\ldots,x_n) = 0$. The <u>maximum satisfiability problem</u> is to find the largest number of logical clauses from the given set for which the satisfiability problem has a solution. We have already described in Section 4.1 a probabilistic approach in paper [13] to the maximum satisfiability problem. The satisfiability problem is notorious for being the original NP-complete problem. One approach has therefore been to study a restricted class of Boolean functions that admit polynomial-time algorithms for the satisfiability problem. A tractable special case of satisfiability arises when the Boolean function is <u>Horn</u>, i.e., each C_i involves at most one complemented variable. Horn functions are well known for their use in rule-based systems and PROLOG. A Boolean function is called a <u>generalized Horn function</u> if it is constructed from a Horn function by disjuncting a nested set of complemented variables to it. A Boolean function in disjunctive normal form is said to be <u>renamable Horn</u> if it is Horn after complementation of some variables. In paper [15], we present succinct mathematical characterizations and linear-time algorithms for recognizing renamable Horn and generalized Horn functions. The algorithm for recognizing renamable Horn functions gives a new method to test for 2-satisfiability.

Logical inference is the central problem in many areas of artificial intelligence, e.g., automated theorem proving, logic progamming, databases, and so on. This problem is usually formulated in a restricted form: given a knowledge-base, i.e., a list of valid facts and rules about atomic assertions, can one infer the validity of <u>some</u> statement involving the atoms. For most applications, it would be easy to argue the importance of a more general version of the inference problem, namely, infer <u>all</u> valid statements implied by the knowledge-base. Such valid implications can be added as new facts or rules to the knowledge-base, thus increasing the efficiency and deductive power of the system in specific applications. Alternatively, valid implications can be used to determine effective question-asking strategies, in the interactive dialogue between an expert system and its human user. The paper [6] investigates the complexity of the general inference problem in the framework of propositional logic. Specifically, it asks: Given a propositional formula in conjunctive normal form, find all prime implications of the formula. Here, a prime implication means a minimal clause whose validity is implied by the validity of the formula. We show that, under some reasonable assumptions, this problem can be solved in time polynomially bounded in the size of the input and in the number of prime implications. In the case of Horn formulae, the result specializes, to yield an algorithm whose complexity grows only linearly with the number of prime implications. The result also applies to a class of formulae generalizing both Horn and quadratic formulae.

An important problem in classical databases is to determine the best logical form of a query in such a database, when the attributes it uses are known. (See Heine [1984, 1986].) In paper [38], this problem is formulated as an unconstrained hyperbolic (or fractional) 0-1 programming problem. The paper then studies unconstrained hyperbolic 0-1 programming problems in general. Such problems are shown to be solvable in linear time when the numerator and the denominator are linear and the latter is always positive. If the denominator is not always positive, they are shown to be NP-hard, and it is even NP-hard to find an approximate solution with a value less than 1/2 larger than the optimal one. Applying the results to the query problem, we show how to solve it in $O(n \log n)$ time, where n is the number of elementary logical conjuncts of the attributes. The linear algorithm derived is shown to provide best queries for various criteria.

If several queries in a database are considered, one may wish to find the form which is best on average. This leads to a new optimization problem, the 0-1 hyperbolic sum problem, which is studied in paper [39]. The problem is shown to be NP-hard and we propose a branch-and-bound algorithm and heuristics of the simulated annealing and tabu search types.

References

Note: References by number, e.g., [3], refer to papers supported by the grant and prepared during the period November 1, 1988 through October 31, 1989. These papers are listed in the next section.

Berge, C., "Färbung von Graphen, deren Sämtliche bzw. deren Ungerade Kreise Starr Sind (Zusammenfassung)," *Wiss. Z. Martin Luther Univ. Math. Natur. Reihe*, 10 (1961), 114-115.

Billera, L.J., "Homology of Smooth Splines: Generic Triangulations and a Conjecture of Strang," *Trans. Amer. Math. Soc.*, 310 (1988), 325-340.

Billera, L.J., "The Algebra of Continuous Piecewise Polynomials over a Simplicial Complex," *Advances in Mathematics*, 1990, to appear.

Bokhari, S.H., *Assignment Problems in Parallel and Distributed Computing*, Kluwer Academic Publishers, Boston, 1987.

Bokhari, S.H., "Partitioning Problems in Parallel, Pipelined and Distributed Computing," *IEEE Trans. Computers*, C37 (1988), 48-57.

Bracken, J., and McGill, J., "Mathematical Programs with Optimization Problems in the Constraints," *Oper. Res.*, 21 (1973), 37-44.

Bracken, J., and McGill, J., "A Method for Solving Mathematical Programs with Nonlinear Programs in the Constraints," *Oper. Res.*, 22 (1974), 1097-1101.

Bracken, J., Falk, J., and McGill, J., "The Equivalence of Two Mathematical Programs with Optimization Problems in the Constraints," *Oper. Res.*, 22 (1974), 1102-1104.

Bradley, G.H., Hammer, P., and Wolsey, L.A., "Coefficient Reduction in 0-1 Programming," *Math. Programming*, 7 (1974), 263-282.

Butz, L., Hammer, P., and Hausmann, D., "Reduction Methods for the Vertex Packing Problem," Tech. Rep. CORR 82-45, University of Waterloo, Waterloo, Ontario, December 1982.

Cameron, P.J., *Parallelisms of Complete Designs*, Cambridge University Press, Cambridge, 1976.

Chvatal, V., "Perfectly Ordered Graphs," in C. Berge and V. Chvatal (eds.), *Topics on Perfect Graphs*, North-Holland, Amsterdam, 1984, 63-65.

Chvatal, V., and Hammer, P., "Aggregation of Inequalities in Integer Programming," *Annals Discr. Math.* 1, (1977), 145-162.

Cohen, J.E., "Interval Graphs and Food Webs: A Finding and a Problem," RAND Corporation Document 17696-PR, Santa Monica, California, 1968.

Cozzens, M., and Leibowitz, R., "Threshold Dimensions of Graphs," *SIAM J. Alg. Discr. Meth.*, 5 (1984), 579-595.

Dembo, R.S., and Hammer, P., "A Reduction Algorithm for Knapsack Problems," *Methods of Operations Res.*, 36 (1980), 49-60.

Dyer, M.E., "The Complexity of Vertex Enumeration Methods," *Math. of Oper. Res.*, 8 (1983), 381-402.

Erdos, P., Ordman, E.T., and Zalcstein, Y., "Bounds on Threshold Dimension and Disjoint Threshold Covering," *SIAM J. Alg. Disc. Meth.*, 8 (1987), 151-154.

Erdos, P., Rubin, A.L., and Taylor, H., "Choosability in Graphs," *Congr. Num.*, 26 (1979), 125-157.

Falk, J.E., and Hoffman, K.L., "Concave Minimization via Collapsing Polytopes," *Oper. Res.*, 34 (1986), 919-929.

Fishburn, P.C., and Roberts, F.S., "Unique Finite Measurement," in F.S. Roberts (ed.), *Applications of Combinatorics and Graph Theory in the Biological and Social Sciences*, Vol. 17 of IMA Volumes in Mathematics and its Applications, Springer-Verlag, New York, 1989, 103-137.

Fortet, R., "L'Algèbre de Boole et ses Applications en Recherche Opérationnelle," *Cahiers du Centre d'Etudes de Recherche Opérationnelle*, 1 (1959), 5-36.

Fortet, R., "Applications de l'Algèbre de Boole en Recherche Opérationnelle," *Revue Francaise d'Informatique et de Recherche Opérationnelle*, 4 (1960), 17-25.

Frankl, P., and Rodl, V., "Near-perfect Coverings in Graphs and Hypergraphs," *Europ. J. Comb.*, 6 (1985), 155-162.

Fulkerson, D.R., and Gross, O.A., "Incidence Matrices and Interval Graphs," *Pac. J. Math.*, 15 (1965), 835-855.

Gale, D., "A Theorem on Flows in Networks," *Pac. J. Math.*, 7 (1957), 1073-1082.

Gallo, G., Grigoriadis, M., and Tarjan, R., "A Fast Parametric Maximum Flow Algorithm," ***SIAM J. Computing***, 18 (1989), 30-55.

Golumbic, M.C., ***Algorithmic Graph Theory and Perfect Graphs***, Academic Press, New York, 1980.

Golumbic, M.C., and Goss, C.F., "Perfect Elimination and Chordal Bipartite Graphs," ***J. Graph Th.***, 2 (1978), 155-163.

Granot, F., and Hammer, P., "On the Role of Generalized Covering Problems," ***Cahiers du Centre d'Etudes de Recherche Operationelle***, 16 (1974), 277-289.

Greenberg, H.J., Lundgren, J.R., and Maybee, J.S., "Graph-Theoretic Foundations of Computer-assisted Analysis," in H.J. Greenberg and J.S. Maybee (eds.), ***Computer Assisted Analysis and Model Simplification***, Academic Press, New York, 1981.

Gyárfás, A., and Lehel, J., "On-line and First-fit Colorings of Graphs," ***J. Graph Th.***, 12 (1988), 217-227.

Hammer, P.L., Hansen, P., and Simeone, B., "Roof Duality, Complementation and Persistence in 0-1 Optimization," ***Math. Programming***, 28 (1984), 121-155.

Hammer, P.L., Ibaraki, T., and Simeone, B., "Threshold Sequences," ***SIAM J. Alg. Discr. Meth.***, 2 (1981), 39-49.

Hammer, P.L., Johnson, E.L., and Peled, U.N., "Regular 0-1 Programs," ***Cahiers du Centre d'etudes de Recherche Operationelle***, 16 (1974), 267-276.

Hammer, P.L., and Mahadev, N.V.R., "Bithreshold Graphs," ***SIAM J. Alg. Disc. Meth.***, 6 (1985), 497-506.

Hammer, P.L., Rosenberg, I., and Rudeanu, S., "On the Determination of the Minima of Pseudo-Boolean Functions," ***Stud. Cerc. Mat.***, 14 (1963), 359-364. (a)

Hammer, P.L., Rosenberg, I., and Rudeanu, S., "Application of Discrete Linear Programming to the Minimization of Boolean Functions," ***Rev. Math. Pure Appl.***, 8 (1963), 459-475. (b)

Hammer, P.L., and Rudeanu, S., ***Boolean Methods in Operations Research and Related Areas***, Springer-Verlag, Berlin/Heidelberg/New York, 1968. (Edition francaise, Dunod, Paris, 1970.)

Hammer, P.L., Simeone, B., Liebling, Th.M., and de Werra, D., "From Linear Separability to Unimodality: A Hierarchy of Pseudo-Boolean Functions," ***SIAM J. Disc. Math.***, 1 (1988), 174-184.

Hansen, P., and Hammer, P., "Logical Relations in Quadratic 0-1 Programming," ***Rev. Roum. Math. Pures et Appl.***, 26 (1981), 421-429.

Hansen, P., Hammer, P.L., and Simeone, B., "On Vertices Belonging to All or to No Maximum Stable Sets of a Graph," *SIAM J. Alg. & Discr. Meth.*, 3 (1982), 511-522.

Hansen, P., Jaumard, B., and Lu, S.H., "A Framework for Algorithms in Globally Optimal Design," *ASME Trans. on Mechanisms, Transmissions and Automation in Design*, 111 (1989), 353-360. (a)

Hansen, P., Jaumard, B., and Lu, S.H., "An Automated Procedure for Globally Optimal Design," *ASME Trans. on Mechanisms, Transmissions and Automation in Design*, 111 (1989), 363-369. (b)

Hansen, P., Jaumard, B., and Lu, S.H., "Some Further Results on Monotonicity in Globally Optimal Design," *ASME Trans. on Mechanisms, Transmissions and Automation in Design*, 111 (1989), 345-352. (c)

Hansen, P., and Kuplinsky, J.. "The Smallest Hard-to-Color Graph," RUTCOR Research Report RRR 5-88, February 1988.

Hansen, P., Labbe, M., Peeters, P., Thisse, J-F., and Henderson, J.V., "Systems of Cities and Facility Location," in *Fundamentals of Pure and Applied Economics*, Vol. 22, Harwood, Chur, 1987.

Heine, M.H., "Information Retrieval from Classical Databases from a Signal Detection Standpoint: A Review," *Information Technology*, 3 (1984), 95-112.

Heine, M.H., "Automatic Optimization Search Logic," Contribution to the Round Table Workshop on Techniques for Access to Information and its Re-Use, November 25-26, 1986, European Economic Communities, Luxembourg.

Henderson, P.B., and Zalcstein, Y., "A Graph-theoretic Characterization of the PV Chunk Class of Synchronizing Primitives," *SIAM J. Computing*, 6 (1977), 88-108.

Hock, W., and Schittkowski, K., "Test Examples for Nonlinear Programming Codes," *Lecture Notes in Economics and Mathematical Systems*, 18, Springer-Verlag, Heidelberg, 1981.

Hoffman, A.J., "Some Recent Applications of the Theory of Linear Inequalities to Extremal Combinatorial Analysis," in Proceedings of Symposia in Applied Mathematics, Vol. X, *Combinatorial Analysis*, American Mathematical Society, Providence, Rhode Island, 1960, 113-127.

Jungnickel, D., and Vanstone, S.A., "Hyperfactorizations of Graphs and 5-designs," *J. Kuwait Univ.*, 14 (Sci.) (1987), 213-224.

Kahn, J., and Saks, M., "Balancing Poset Extensions," *Order*, 1 (1984), 113-126.

Lundgren, J.R., "Food Webs, Competition Graphs, Competition-Common Enemy Graphs, and Niche Graphs," in F.S. Roberts (ed.), *Applications of Combinatorics and Graph Theory to the Biological and Social Sciences*, Volume 17 of IMA Volumes in Mathematics and its Applications, Springer-Verlag, New York, 1989, 221-243.

Maimon, O., "The Variance Equity Measures in Location Decision Theory," *Annals of Oper. Res.*, 6 (1986), 147-160.

Maimon, O., "An Algorithm for the Lorenz Measure in Locational Decisions on Trees," *J. of Algor.*, 9 (1988), 583-596.

Meyniel, H., "On the Perfect Graph Conjecture," *Discr. Math.*, 16 (1976), 334-342.

Opsut, R.J., "On the Computation of the Competition Number of a Graph," *SIAM J. Alg. & Discr. Meth.*, 3 (1982), 420-428.

Opsut, R.J., and Roberts, F.S., "On the Fleet Maintenance, Mobile Radio Frequency, Task Assignment, and Traffic Phasing Problems," in G. Chartrand, Y. Alavi, D.L. Goldsmith, L. Lesniak-Foster, and D.R. Lick (eds.), *The Theory and Applications of Graphs*, Wiley, New York, 1981, 479-492.

Peled, U.N., and Simeone, B., "The Threshold Measure of a Graph," Report RRR 9-87, Rutgers Center for Operations Research, February 1987.

Pippenger, N., and Spencer, J., "Asymptotic Behavior of the Chromatic Index for Hypergraphs," *J. Comb. Th.*, A51 (1989), 24-42.

Prekopa, A., "Boole-Bonferroni Inequalities and Linear Programming," *Oper. Res.*, 36 (1988), 145-162.

Prekopa, A., "Sharp Bounds on Probabilities Using Linear Programming," *Oper. Res.*, 1990, to appear.

Provan, J.S., "Determinacy in Linear Systems and Networks," *SIAM J. Alg. & Discr. Meth.*, 4 (1983), 262-278.

Provan, J.S., and Kydes, A., "Correlation and Determinacy in Network Models," BNL Rep. 51243, Brookhaven National Laboratory, Upton, New York, 1980.

Raychaudhuri, A., and Roberts, F.S., "Generalized Competition Graphs and their Applications," in P. Brucker and R. Pauly (eds.), *Methods of Operations Research*, Vol. 49, Anton Hain, Konigstein, West Germany, 1985, 295-311.

Roberts, F.S., "Food Webs, Competition Graphs, and the Boxicity of Ecological Phase Space," in Y. Alavi and D. Lick (eds.), *Theory and Applications of Graphs*, Springer-Verlag Lecture Notes #642, Berlin/Heidelberg/New York, 1978, 477-490.

Roberts, F.S., *Measurement Theory, with Applications to Decisionmaking, Utility, and the Social Sciences*, Addison-Wesley, Reading, Mass., 1979. (a)

Roberts, F.S., "On the Mobile Radio Frequency Assignment Problem and the Traffic Light Phasing Problem," *Annals N.Y. Acad. Sci.*, 319 (1979), 466-483. (b)

Roberts, F.S., *Applied Combinatorics*, Prentice-Hall, Englewood Cliffs, N.J., 1984.

Roberts, F.S., "Applications of the Theory of Meaningfulness to Psychology," *J. Math. Psychol.*, 29 (1985), 311-332. (a)

Roberts, F.S., "Issues in the Theory of Uniqueness in Measurement," in I. Rival (ed.), *Graphs and Orders*, Reidel, Amsterdam, 1985, 415-444. (b)

Roberts, F.S., "Meaningless Statements, Matching Experiments, and Colored Digraphs (Applications of Graph Theory and Combinatorics to the Theory of Measurement)," in F.S. Roberts (ed.), *Applications of Combinatorics and Graph Theory to the Biological and Social Sciences*, Volume 17 of IMA Volumes in Mathematics and its Applications, Springer-Verlag, New York, 1989, 277-294.

Robertson, N., and Seymour, P.D., "Graph Minors. II. Algorithmic Aspects of Tree-Width," *J. of Algo.*, 7 (1986), 309-322.

Scott, D.D., "The Competition-Common Enemy Graph of a Digraph," *Discr. Appl. Math.*, 17 (1987), 269-280.

Shannon, C.E., "The Zero-error Capacity of a Noisy Channel," *IRE Trans. Inform. Theory*, IT-2 (1956), 8-19.

Sheridan, T.B., and Ferrell, W.R., *Man-Machine Systems: Information, Control and Decision Models of Human Performance*, MIT Press, Cambridge, Mass., 1974.

Thach, P.T., and Tuy, H., "Global Optimization under Lipschitzian Constraints," *Japan J. Appl. Math.*, 4 (1987), 205-217.

Tuy, H., "A General Deterministic Approach to Global Optimization via d.c. Programming," in J.-B. Hiriart-Urruty (ed.), *Fermat Days 85: Mathematics for Optimization*, North-Holland, Amsterdam, 1986, 273-303.

Zemel, E., "Measuring the Quality of Approximate Solutions to Zero-One Programming Problems," *Math. of Oper. Res.*, 6 (1981), 319-332.

A RUTCOR Project in Discrete Applied Mathematics

Grant Number AFOSR 89-0066

List of Publications

November 1, 1988 - October 31, 1989

This list contains papers prepared or worked on during the period of the grant. The abbreviation RRR stands for RUTCOR Research Report.

1. Bagchi, A., and Williams, A.C., "On the Number of Local Maxima for 0-1 Quadratic Programs," in preparation.

2. Bagchi, A., and Williams, A.C., "Recognition Problems for Certain Classes of Pseudo-Boolean Functions," RRR 43-89, October 1989.

3. Barthelemy, J., Hudry, O., Isaak, G., Roberts, F.S., and Tesman, B., "On the Reversing Number of a Digraph," in preparation.

4. Billera, L.J., and Rose, L.L., "Groebner Basis Methods for Multivariate Splines," RRR 1-89, January 1989.

5. Blidia, M., Duchet, P., and Maffray, F., "On the Orientation of Meyniel Graphs," RRR 3-90, January 1990, in press.

6. Boros, E., Crama, Y., and Hammer, P.L., "Polynomial-Time Inference of all Valid Implications for Horn and Related Formulae," RRR 22-89, April 1989. To appear in *Annals of A.I.*

7. Boros, E., Crama, Y., and Hammer, P.L., "Upper Bounds for Quadratic 0-1 Maximization Problems," RRR 14-89, March 1989. In press, *Operations Research Letters*.

8. Boros, E., and Hammer, P.L., "A Max-Flow Approach to Improved Roof Duality in Quadratic 0-1 Minimization," RRR 15-89, March 1989.

9. Boros, E., Hammer, P.L., and Sun, X., "The DDT Method for Quadratic 0-1 Minimization," RRR 39-89, October 1989.

10. Boros, E., Jungnickel, D., and Vanstone, S.A., "The Existence of Non-trivial Hyperfactorizations of K_{2n}," *Combinatorica*, to appear.

11. Boros, E., and Prekopa, A., "Availability Analysis and the

Method of Binomial Moments to Evaluate Reliability of Power Systems," *Proc. of Resource Planning Under Uncertainty for Electric Power Systems*, in press.

12. Boros, E., and Prekopa, A., "On the Probability of the Existence of a Feasible Flow in a Transportation Network," *Operations Research*, to appear.

13. Boros, E., and Prekopa, A., "Probabilistic Bounds and Algorithms for the Maximum Satisfiability Problem," *Annals of Oper. Res.*, in press.

14. Bourjolly, J-M., Hammer, P.L., Pulleyblank, W., and Simeone, B., "Combinatorial Methods for Bounding Quadratic Pseudo-Boolean Functions," RRR 27-89, August 1989.

15. Chandru, V., Coullard, C.R., Hammer, P.L., Montanez, M., and Sun, X., "On Renamable Horn and Generalized Horn Functions," RRR 44-89, October 1989.

16. Chen, P., Hansen, P., and Jaumard, B., "On-line Vertex Enumeration by Adjacency Lists," in preparation.

17. Crama, Y., Hansen, P., and Jaumard, B., "The Basic Algorithm for Pseudo-Boolean Programming Revisited," RRR 54-88, November 1988.

18. de Werra, D., and Roberts, F.S., "On the Use of Augmenting Chains in Chain Packings," *Discr. Appl. Math.*, to appear.

19. Ding, G., "Monotone Clutters," RRR 52-89, November 1989.

20. Ding, G., "Order the Vertices of a Graph Linearly," RRR 53-89, December 1989.

21. Ding, G., and Wang, C., "Threshold Digraphs," RRR 50-89, November 1989.

22. Furedi, Z., Kahn, J., and Kleitman, D.J., "Sphere Coverings of the Hypercube with Incomparable Centers," *Discr. Math.*, to appear.

23. Hammer, P.L., and Maffray, F., "Preperfect Graphs," RRR 35-89, September 1989.

24. Hammer, P.L., Maffray, F., and Preissmann, M., "A Characterization of Chordal Bipartite Graphs," RRR 16-89, April 1989.

25. Hammer, P.L., Mahadev, N.V.R., and Peled, U.N., "Bipartite Bithreshold Graphs," RRR 13-89, February 1989.

26. Hammer, P.L., Peled, U.N., and Sun, X., "Difference Graphs," RRR 12-89, February 1989.

27. Hanjoul, P., Hansen, P., Peeters, D., and Thisse, J., "Uncapacitated Plant Location under Alternative Spatial Price Policies," RRR 61-88, December 1988. In *Management Sci.*, 36 (1990), 41-57.

28. Hansen, P., Jaumard, B., and Frank, O., "An $O(N^2)$ Algorithm for Maximum Sum-of-Splits Clustering," RRR 10-89, February 1989. In *J. of Classification*, 6 (1989), 177-193.

29. Hansen, P., Jaumard, B., and Lu, S., "An Analytical Approach to Global Optimization," RRR 4-89, January 1989. To appear in *Math. Progr.*

30. Hansen, P., Jaumard, B., and Lu, S., "Global Optimization of Univariate Lipschitz Functions: I. Survey and Properties," RRR 18-89, April 1989.

31. Hansen, P., Jaumard, B., and Lu, S., "Global Optimization of Univariate Lipschitz Functions: II. New Algorithms and Computational Comparison," RRR 23-89, May 1989.

32. Hansen, P., Jaumard, B., and Mathon, V., "Constrained Nonlinear 0-1 Programming," RRR 47-89, November 1989.

33. Hansen, P., Jaumard, B., and Musitu, K., "Weighted Constrained Maximum Split Clustering," RRR 28-89, August 1989.

34. Hansen, P., Jaumard, B., and Savard, G., "A Variable Elimination Algorithm for Bilevel Linear Programming," RRR 17-89, April 1989.

35. Hansen, P., and Kuplinsky, J., "Finding Small Hard-to-Color Graphs by Computer," in preparation.

36. Hansen, P., and Lih, K., "Improved Algorithms for Partitioning Problems in Parallel, Pipelined and Distributed Computing," RRR 38-89, October 1989.

37. Hansen, P., Peeters, D., and Thisse, J-F., "Plant Location for Profit Maximization with a Uniform Delivered Price per Zone Policy," in preparation.

38. Hansen, P., Poggi de Aragao, M.V., and Ribeiro, C.C., "Hyperbolic 0-1 Programming and Queries Optimization in Classical Databases," RRR 25-89, June 1989.

39. Hansen, P., Poggi de Aragao, M.V., and Ribeiro, C.C., "The 0-1 Hyperbolic Sum Problem: Algorithms and an Application to Queries Optimization," in preparation.

40. Hansen, P., and Zheng, M., "Algorithms for Determining a Lorenz Point on Trees and Networks," RRR 55-89, December 1989.

41. Hansen, P., and Zheng, M., "Minimizing the Variance of Distances on a Network," in preparation.

42. Hansen, P., and Zheng, M., "The Shortest Shortest Path Tree of a Network," in preparation.

43. Harary, F., Kim, S., and Roberts, F.S., "Extremal Competition Numbers as a Generalization of Turan's Theorem," RRR 6-89, January 1989. To appear in *J. Ramanujan Mathematical Society*.

44. Hertz, A., Hansen, P., Jaumard, B., and Kuplinsky, J., "Minimum Diameter Weight-Constrained Clustering," in preparation.

45. Hoang, C., and Maffray, F., "On Slim Graphs, Even Pairs, and Star-Cutsets," RRR 55-88, November 1988.

46. Hoang, C., Maffray, F., and Preissmann, M., "New Properties of Perfectly Orderable Graphs and Strongly Perfect Graphs," RRR 26-89, August 1989.

47. Isaak, G., Kim, S., McKee, T.A., McMorris, F.R., and Roberts, F.S., "2-Competition Graphs," RRR 2-90, in press.

48. Kahn, J., "A Problem of Erdos and Lovasz," in preparation.

49. Kahn, J., "Coloring Nearly-disjoint Hypergraphs with $n+o(n)$ Colors," submitted.

50. Kahn, J., "On a Theorem of Frankl and Rodl, part I," in preparation.

51. Kahn, J., "On a Theorem of Frankl and Rodl, part II," in preparation.

52. Kahn, J., Komlos, J., and Szemeredi, E., "Singularity Probabilities for Random $\{\pm 1\}$-matrices," in preparation.

53. Kahn, J., and Linial, N., "Balancing Extensions via Brunn-Minkowski," *Combinatorica*, to appear.

54. Kahn, J., and Meshulam, R., "On Mod p Transversals," RRR 5-89, January 1989. To appear in *Combinatorica*.

55. Kahn, J., and Seymour, P.D., "A Fractional Version of the Erdos-Faber-Lovasz Conjecture," RRR 37-89, October 1989. To appear in *Combinatorica*.

56. Kahn, J., and Szemeredi, E., "The Second Eigenvalue of a Random Regular Graph," preliminary version. (Short version, with J. Friedman as additional author, appeared as first chapter in 21st STOC, ACM, 1989.)

57. Kim, S., McKee, T.A., McMorris, F.R., and Roberts, F.S., "p-Competition Graphs," RRR 36-89, October 1989.

58. Kim, S., and Roberts, F.S., "On Opsut's Conjecture about the Competition Number," RRR 20-89, April 1989. To appear in *Congr. Num.*

59. Kim, S., Roberts, F.S., and Seager, S., "On 1 0 1 - Clear (0,1) Matrices and the Double Competition Number of Bipartite Graphs," RRR 19-89, April 1989.

60. Lu, S., *Essays on Global Optimization - Theory and Algorithms*, Ph.D. Thesis, RUTCOR, October 1989. Published in RUTCOR Dissertation series as RD 2-89, October 1989.

61. Maffray, F., *Structural Aspects of Perfect Graphs*, Ph.D. Thesis, RUTCOR, October 1989. Published in RUTCOR Dissertation series as RD 3-89, October 1989.

62. Peled, U.N., and Sun, F., "Enumeration of Labeled Difference Graphs and an Identity of Stirling Numbers," RRR 56-89.

63. Roberts, F.S., "Applications of Combinatorics and Graph Theory to the Biological and Social Sciences: Seven Fundamental Ideas," RRR 21-89, May 1989. In F. S. Roberts (ed.), *Applications of Combinatorics and Graph Theory to the Biological and Social Sciences*, Volume 17 of IMA Volumes in Mathematics and its Applications, Springer Verlag, New York, 1989, pp. 1-37.

64. Roberts, F.S., "Meaningfulness of Conclusions from Combinatorial Optimization," *Discr. Appl. Math.*, to appear.

65. Tesman, B.A., "Application of Forbidden Difference Graphs to T-Colorings," mimeographed, March 1989.

66. Tesman, B.A., *T-Colorings, List T-Colorings, and Set T-Colorings of Graphs*, Ph.D. Thesis, Department of Mathematics, Rutgers University, October 1989. In press as RRR 57-89.

67. Wang, C., and Williams, A.C., "The Threshold Order of a Boolean Function," RRR 62-88, December 1988. To appear in *Discr. Appl. Math.*

68. Wang, C., and Williams, A.C., "The Threshold Weight of a Graph," RRR 32-89, September 1989.

69. Williams, A.C., "Estimating the Numbers of Threshold Functions of Dimension n and Order m," RRR 24-89, May 1989.

70. Zheng, M., and Lu, X., "On the Maximum Induced Forest of a Connected Cubic Graph without Triangles," RRR 8-89, January 1989.

A RUTCOR Project in Discrete Applied Mathematics

Grant Number AFOSR 89-0066

Lectures Delivered

November 1, 1988 - October 31, 1989

Peter L. Hammer

"Mathematics and Artificial Intelligence," introductory remarks at Workshop on Mathematics and Artificial Intelligence, Ulm, Germany, December 1988. (Dr. Hammer was Workshop co-director.)

"Cause Effect Relationships and Partially Defined Boolean Functions," Yale School of Organization and Management, New Haven, Connecticut, April 1989.

"Some Properties of 2-threshold Graphs," invited lecture, NATO Advanced Research Workshop on Topological Network Design, Denmark, June 1989.

"Logic and Operations Research," introductory remarks at Workshop on Logic and Operations Research, Ulm, Germany, September 1989. (Dr. Hammer was Workshop co-director.)

"The DDT Method for Quadratic 0-1 Minimization," invited lecture, 14th Symposium on Operations Research, Ulm, Germany, September 1989.

Fred S. Roberts

"Applications of Discrete Mathematics," 4-lecture minicourse at National Meeting of Mathematical Association of America, Phoenix, Arizona, January 1989.

Titles of Individual Lectures:

"Applications of Graph Coloring"
"Applications of Eulerian Chains and Paths"
"Applications of Simple Counting Rules"
"Applications of Generating Functions"

"From Garbage to Rainbows: The Many Applications of Graph Coloring," colloquium talk, Glassboro State College, Glassboro, NJ, January 1989.

"The One-Way Street Problem," all-departmental seminar talk, Johns Hopkins University, Baltimore, Maryland, February 1989.

"On Opsut's Conjecture for Competition Graphs," contributed paper at Southeastern Conference on Combinatorics, Graph Theory, and Computing, Boca Raton, Florida, February 1989.

"From Garbage to Rainbows: The Many Applications of Graph Coloring," keynote address to 4th Las Cruces Symposium on Graph-Theoretic Methods (Applications of Graph Theory to Computer Science and Chemistry), New Mexico State University, Las Cruces, New Mexico, March 1989.

"Discrete Mathematics," half-day workshop for faculty at Westfield High School, Westfield, NJ, March 1989.

"Applications of Graph Theory," featured 2-lecture series at 6th Annual Rose-Hulman Conference on Undergraduate Mathematics, Rose-Hulman Technological Institute, Terre-Haute, Indiana, April 1989.

Titles of individual lectures:

"The One-Way Street Problem"
"From Garbage to Rainbows"

"Discrete Mathematics," invited plenary lecture at Hartford Conference on Teaching the Mathematical Core, University of Hartford, Hartford, Connecticut, April 1989.

"The One-Way Street Problem," colloquium, Supercomputing Research Center, Bowie, Maryland, May 1989.

"Letters, Mobile Radio Telephones, and Rainbows: The Many Applications of Modern Applied Mathematics," invited talk, Rahway High School, Rahway, NJ, May 1989.

"From Rainbows to Ham and Cheese Sandwiches: T-Colorings of Graphs and their Applications," colloquium talk, Wright State University, Dayton, Ohio, May 1989.

"Graphs, Garbage, and a Pollution Solution," invited lecture to Douglass Science Institute for High School Girls, Douglass College, New Brunswick, NJ, June 1989.

"Applications of Graph Coloring," 5-lecture series at DIMACS Summer Workshop for High School Teachers, July 1989.

Titles of Individual Lectures:

"The One Way Street Problem"
"Applications of Graph Coloring"
"Applications of Eulerian Chains and Paths"
"Applications of Simple Counting Rules I"
"Applications of Simple Counting Rules II"

"From Radios to Rainbows: The Many Applications of Graph Coloring," invited talk to the Air Force Office of Scientific

Research Command at AFOSR Day, Bolling Air Force Base, DC, August 1989.

"The One-Way Street Problem," colloquium talk, University of Puerto Rico, San Juan, Puerto Rico, August 1989.

"The One-Way Street Problem," colloquium talk, Williams College, Williamstown, Massachusetts, October 1989.

Endre Boros

"Graph Balancing," contributed paper, Southeastern Conference on Graph Theory, Combinatorics, and Computing, Boca Raton, Florida, February 1989.

"Rectangular Subdivisions," at Optimization Days meeting, Montreal, May 1989.

"Persistency in Quadratic 0-1 Programming," invited talk at combined national meeting of Canadian Operations Research Society, The Institute of Management Science, and the Operations Research Society of America, Vancouver, May 1989.

"Chvatal Closure of Standard Linearizations," EURO X (Annual Meeting of European Operations Research Societies), Beograd, June 1989.

"On Random Knapsack Problems," invited talk, national meeting of The Institute of Management Sciences/The Operations Research Society of America, New York, October 1989.

Guoli Ding

"2-Monotone Matroids, 3-Monotone Matroids, and Threshold Matroids," talk at Advanced Research Institute in Discrete Applied Mathematics, Rutgers University, New Brunswick, NJ, May 1989.

Pierre Hansen

"A Variable Elimination Method for Bilevel Linear Programming," invited talk at Workshop on Numerical Aspects of Combinatorial Optimization, Oberwolfach, January 1989.

"A Dozen Approaches to Global Optimization" inaugural plenary lecture, Optimization Days conference, Montreal, May 1989.

"Maximizing the Product of Linear 0-1 Functions," invited lecture at CORS-ORSA-TIMS combined national meeting, Vancouver, May 1989.

"A Sequential Polynomial Approximation Algorithm for Global Optimization of Univariate Functions," invited lecture at

CORS-ORSA-TIMS combined national meeting, Vancouver, May 1989.

"An Analytical Approach to Global Optimization," colloquium, CORE, University of Louvain, Belgium, May 1989.

"Sharp Bounds on the Order, Size, and Stability Number of a Graph," invited talk at Fourth Advanced Research Institute in Discrete Applied Mathematics, Rutgers University, May 1989.

"The Basic Algorithm of Pseudo-Boolean Programming Revisited," colloquium, University of Puerto Rico, San Juan, June 1989.

"Quelques Approches de l'Optimisation Globale," colloquium, Conservatoire National des Art et Metiers, Paris, June 1989.

"Global Optimization of Univariate Lipschitz Functions, I and II," at EURO X (annual meeting of European Operations Research Societies), Beograd, June 1989.

"Global Optimization of Univariate Lipschitz Functions," plenary lecture, International School - Seminar on Optimization Methods and their Applications, Irkutsk, Siberia, USSR, September 1989.

"Global Optimization of Univariate Lipschitz Functions, I: Survey and Properties," invited lecture at ORSA-TIMS national meeting, New York, October 1989.

"Tabu Search and Probabilistic Satisfiability," plenary lecture at FNRS conference on A New Approach to Combinatorial Optimization: Tabu Search, Brussels, Belgium, October 1989.

Jeffry Kahn

"On a Problem of Erdos and Lovasz and Related Matters," University of Delaware, April 1989.

"Singularity Problems for Random 0-1 Matrices," University of Pennsylvania, Philadelphia, May 1989.

"Influence of Variables on Boolean Functions," AT&T Bell Laboratories, Murray Hill, NJ, August 1989.

Barry Tesman

"T-Colorings of Graphs and the Channel Assignment Problem," seminar, Xavier University, Cincinnati, Ohio, January 1989.

"T-Colorings of Graphs and their Applications," seminar, Randolph-Macon College, January 1989.

"T-Colorings of Graphs and the Channel Assignment Problem," seminar, Dickinson College, Carlisle, PA, January 1989.

"T-Colorings of Graphs and the Channel Assignment Problem," seminar, Worcester Polytechnic Institute, Worcester, MA, February 1989.

"Application of Forbidden Difference Graphs to T-Coloring," Southeast Conference on Graph Theory, Combinatorics, and Computing, Boca Raton, Florida, February 1989.

Albert Williams

"Extrapolating Boolean Functions using Generalized Threshold Functions," Fourth Advanced Research Institute in Discrete Applied Mathematics, Rutgers University, May 1989.

Participants in "A RUTCOR Project in Discrete Applied Mathematics"

November 1, 1988 - October 31, 1989

FACULTY

Peter Hammer (Principal Investigator)

Fred Roberts (Principal Investigator)

Louis Billera

Endre Boros

Vasek Chvatal

Pierre Hansen

Jeffry Kahn

Michael Saks

Albert Williams

POSTDOCTORAL FELLOW

Endre Boros

GRADUATE STUDENTS

Ansuman Bagchi

Guoli Ding

Janice Kim

Jian-min Long

Shi-Hui Lu

Frederic Maffray

Xiaorong Sun

Barry Tesman

Chi Wang

ASSOCIATE FELLOWS

Claude Benzaken, University of Science and Medicine, Grenoble, France

Yves Crama, University of Limburg, The Netherlands

Brigitte Jaumard, GERAD and Ecole Polytechnique de Montreal

Gil Kalai, Hebrew University

Uri Peled, University of Illinois, Chicago

Dominique de Werra, Swiss Federal Institute of Technology, Lausanne

Jacques Thisse, CORE, University of Louvain, Belgium

ADVISORY COMMITTEE

Egon Balas, Carnegie-Mellon University

Claude Berge, CNRS, Paris

Ronald Graham, AT&T Bell Laboratories

Laszlo Lovasz, Eotvos University and Princeton University

APPENDIX I

RUTCOR SEMINARS
Fall 1988

Date	Speaker	Time Place	Title
07/01/88	Sylvia Halasz Zurich	1:30-2:30 Hill 705	Inventory control system
09/27/88	Reiner Horst Trier University West Germany	1:30-2:30 p.m. Hill 423	Algorithms for concave minimization problems
10/04/88	Henryk Galina Technical University Wroclaw, Poland	1:30-2:30 p.m. Hill 705	Kirchoff Matrices in Polymer Physics
10/11/88	Joseph Zaks University of Calgary Calgary, Canada	1:30-2:30 p.m. HILL 705	On Some Geometric Graphs
10/18/88	Eric V. Denardo Yale University	1:30-2:30 p.m. Hill 705	Pulling a Markov Production System
10/27/88	Silvano Martello DEIS - Univ. Bologna Italy	5:30-6:30 Hill 705 Thursday	Knapsack Problems: Algorithms and Computer Implementations
11/01/88	Albert Belostotsky Philadelphia	Hill 705 1:30-2:30 p.m.	Decision making under uncertainty: a case of data improvement
11/08/88	Robert E. Machol U. S. Dept.of Transp. FAA	Hill 423 1:30-2:30 p.m.	Designing the Aviation System for the 21st Century
11/15/88	Hans Schneider University of Wisconsin	1:30-2:30 p.m. Hill 705	Network flows and matrix scaling
11/22/88	Uriel Rothblum RUTCOR	Hill 705 1:30-2:30 p.m.	Scaling via optimization
11/29/88	Fabio Tardella RUTCOR	Hill 705 1:30-2:30 pm	Vertices of a polyhedron
12/09/88	Eva Tardos & David Schmoys MIT	Hill 423 1:00-3:30 p.m.	Arbitrage problem/ Scheduling problems

RUTCOR SEMINARS
Fall 1988

Date	Speaker	Time Place	Title
12/12/88	Daniel Bienstock Bellcore	1:30-2:30 Hill 705	Planar graphs ...
12/13/88	Kurt Anstreicher Yale University	Hill 705 1:30-2:30	Long steps in a O(n^3L) algorithm
12/20/88	Sjur Flam Inst. of Economics Univ. of Bergen, Norway	1:30-2:30 Hill 705	Finite convergence

RUTCOR SEMINARS
Spring 1989

Date	Speaker	Time Place	Title
01/24/89	Eitan Zemel Northwestern University	Tuesday Hill 705 1:30-2:30	On the complexity of lifting
01/31/89	John Franco Indiana University	Tuesday Hill 705 1:30-2:30	Probabilistic analysis of algorithms ...
02/03/89	Stephan Foldes GERAD	Friday Hill 705 11-12am	Computational Geometry and partial orders: separability problems in the plane
02/06/89	Bruce Reed Waterloo	Monday Hill 705 1:30-2:30	Some Aplications of Lovasz's Local Lemma
02/07/89	Pradeep Shah School of Business Rutgers University	Tuesday Hill 705 2:00-3:00	Decision Problems in mini-load automatic warehousing systems
02/07/89	C.F. Lee School of Business Rutgers	Tuesday Hill 705 1:00-2:00	Interaction of investment, financing and dividend decision a control theory approach
02/20/89	Imre Barany Yale Univ. Hung.Acad.	Monday Hill 423 1:30-2:30	Convex bodies, economic cap coverings, random polytopes
02/21/89	Panos Pardalos Pennsylvania State University	Tuesday Hill 705 1:30-2:30	Computational results of parallel branch and bound algorithms for the maximum clique problem
02/27/89	M.R. Emamy University of Puerto Rico	Monday Hill 705 1:30-2:30	On the complexity of some greedy algorithms over the class of threshold pseudo-Boolean functions
02/28/89	Peter Frankl CNRS & AT&T Bell	Tuesday Hill 705 1:30-2:30	Cross intersecting families
03/08/89	Gyula O.H. Katona Case Western University Cleveland	Wednesday Hill 705 10:30-11:30	Supporting hyperplanes of the class of intersecting families

RUTCOR SEMINARS
Spring 1989

Date	Speaker	Time Place	Title
03/14/89	Michel Troyon RUTCOR	Tuesday Hill 705 1:30-2:30	A new heuristic for the minimum perfect matching problem in the Euclidean plane
03/20/89	Roger J-B. Wets University of California-Davis	Monday Hill 705:1:00-2:00	Approximation theory ..
03/21/89	Marc Teboulle University of Maryland	Tuesday Hill 705 1:30-2:30	Nondifferentiable optimization via smoothing and approximation
03/28/89	Patrick T. Harker The Wharton School UNiversity of Pennsylvania	Tuesday Hill 705 1:30-2:30	Globally convergent methods ...
04/04/89	Jeremy Bloom General Public Utilities Parsippany NJ	Tuesday Hill 705 1:30-2:30	An analytic representation of the production cost curve of a power system and several of its applications
04/18/89	L.K. Jones Dept. Math. University of Lowell, MA.	Tuesday Hill 705 1:30-2:30	Entropy, directed orthogonality, and magic distances
04/18/89	Garth McCormick George Washington University	Tuesday Hill 705 2:30-3:30	Unary convex optimization
04/19/89	Alfredo N. Iusem IMPA, Rio de Janeiro	Wednesday Hill 705 10:30-11:30	Iterative ...
04/24/89	Jaroslav Nesetril Prague & Chicago	Monday Hill 705 1:30-2:30	Ramsey Problem
04/25/89	Takayuki Hibi MIT	Tuesday Hill 705 1:30-2:30	The Ehrhart polynomial ...
05/02/89	Aharon Ben Tal Technion Haifa Israel	Tuesday Hill 705 1:30-2:30	Topics in nonsmooth optimization

RUTCOR SEMINARS
Spring 1989

Date	Speaker	Time Place	Title
05/03/89	Ron Shamir Tel Aviv University	Wednesday Hill 705 10:30-11: 30	Greediness pays in transportation
05/16/89	Raghu Ram UPS	Tuesday Hill 705 1:30-2:30	Opns. Res. at UPS
05/16/89	Cheng Lin U.P.S	2:30-3:30 Hill 705	O.R. at UPS

RUTCOR SEMINARS
Fall 1989

Date	Speaker	Time Place	Title
8/31/89	Tamas Szantai ELTE, Budapest	1:30–2:30 Hill 705	Numerical evaluation of some special multivariate probability distribution functions
9/22/89	Bernhart Fruhwirth Graz	1:30–2:30 Hill 525	Approximation of convex functions with applications to bicriterion linear programming
10/19/89	Eli Shamir Jerusalem	4:30–5:50 Hill 705	Learning a concept class by examples, via quantization of underlying distribution
10/24/89	Uriel Rothblum RUTCOR	1:30–2:30 Hill 705	Time-sharing marriages and polyhedral combinatorics
10/30/89	Janos Fulop CAIHAS	8:30–9:30 Hill 116	A finite cutting plane method for solving linear programs with an additional reserve convex constraint
11/1/89	Paul Erdos Hung. Acad. Sci.	11:30–12:30 Hill 525	Combinatorial problems in geometry
11/3/89	Brigitte Jaumard GERAD	1:00–2:00 7th Lounge	Contrained nonlinear 0–1 programming
11/14/89	Michael Schneider Johns Hopkins	1:30–2:30 Hill 705	Max-balanced flows
11/21/89	M. Rothkopf RUCTOR	3:00–4:00 Hill 703	Enriched modeling of oral auctions
12/04/89	Aart Blokhuis Technical University Eindhoven	1:00–2:00 Hill 705	Problems and results on pointsets in finite projective planes
12/5/89	Erhan Cinlar Princeton University	1:30–2:30 Hill 705	Birth and death on flows
12/11/89	Ulrich Dieter Technical University Graz	1:00–2:00 Hill 705	The greedy and threshold algorithm for the binary knapsack problem: a comparison based on probabilistic assumptions

RUTCOR SEMINAR
Fall 1989

12/12/89	Uri Peled U. of Illinois Chicago	1:30–2:30 Hill 705	Enumeration of difference graphs
12/14/89	Stephan Olariu Old Dominion U.	12:30–1:30 Hill 116	An NC recognition algorithm for cographs

RUTGERS

[illegible] Research
Hill Center for the Mathematical Sciences
New Brunswick · New Jersey 08903

ARIDAM IV

FOURTH ADVANCED RESEARCH INSTITUTE ON DISCRETE APPLIED MATHEMATICS

RUTCOR, Hill Center, Room 705

Opening Day Program for May 30, 1989

9:00-9:30 Registration and Coffee

Morning Session

Chairman: Professor Fred S. Roberts, RUTCOR

9:30-10:00 Welcome and Opening Remarks

Professor T. Alexander Pond, Executive Vice President and Chief Academic Officer, Rutgers University

Dr. Neal Glassman, Program Manager, Mathematics and Information Sciences, Air Force Office of Scientific Research

Professor Peter L. Hammer, Director, RUTCOR

10:00-11:00 Professor Rolf H. Möhring, Technische Universität, Berlin "Graph Problems Related To Gate Matrix Layout And PLA Folding"

11:00-11:20 Coffee Break

11:20-12:20 Professor William T. Trotter, Arizona State University "The Dimension of Planar Maps – the Two Connected Case"

12:20-2:00 Luncheon

Afternoon Session

Chairman: Professor Pierre Hansen, RUTCOR

2:00-3:00 First lecture by Professor Richard M. Karp, University of California, Berkeley, "A Survey of Randomized Algorithms"

3:00-3:30 Coffee Break

3:30-4:30 Second lecture by Professor Richard M. Karp

RUTCOR, Hill Center, Room 705

Program for May 31, 1989

9:00-10:00 Breakfast

Morning Session

Chairman: Professor Fred Hoffman, Florida State University

10:00-12:20 Third and Fourth lectures by Professor Richard M. Karp, University of California at Berkeley, "A Survey of Randomized Algorithms"

12:20-4:00 Period reserved for lunch and free discussions among the participants

Afternoon Session

Chairman: Professor J. Richard Lundgren, University of Colorado at Denver

4:00-4:15 Professor Thomas A. Feo, University of Texas at Austin
"Greedy Randomized Adaptive Search Procedure"

4:15-4:30 Dr. Jens Gustedt, Technische Universität Berlin
"Gate Matrix Layout Problem For Triangulated Graphs Is NP-Complete"

4:30-4:45 Professor Thomas J. Marlowe, Seton Hall University
"Incremented Data Flow Algorithms And Incremented Iteration"

4:45-5:00 Professor Panos Pardalos, The Pennsylvania State University
"Computational Aspects of An Algorithm For The Maximum Weighted Independent Set Problem"

5:00-5:20 Coffee Break

5:20-5:35 Dr. Eric Taillard, Ecole Polytechnique Fédérale de Lausanne
"Parallel Taboo Search For The Flow Shop Sequencing Problem"

5:35-5:50 Dr. Dorothea Wagner, Technische Universität Berlin
"Area-optimal Channel Routing in Log Parallel Time"

5:50-6:05 Professor Albert C. Williams, Rutgers University
"Extrapolating Boolean Functions Using Generalized Threshold Functions"

6:05-6:20 Professor Eitan Zemel, Northwestern University
"Complexity of Game Concepts"

THE STATE UNIVERSITY OF NEW JERSEY
RUTGERS

RUTCOR · Rutgers Center for Operations Research
Hill Center for the Mathematical Sciences
New Brunswick · New Jersey 08903

ARIDAM IV

FOURTH ADVANCED RESEARCH INSTITUTE ON DISCRETE APPLIED MATHEMATICS

RUTCOR, Hill Center, Room 705

Program for Thursday, June 1, 1989 (Revised)

9:00-10:00 Breakfast

Morning Session

Chairman: Professor Thomas J. Marlowe, Seton Hall University

10:00-12:20 Fifth and sixth lectures by Professor Richard M. Karp
University of California at Berkeley
"Combinatorial Aspects of Parallel Computation"

12:20-4:00 Period reserved for lunch and free discussions among the participants

Afternoon Session

Chairman: Professor Aviezri S. Fraenkel, Weizmann Institute of Science

4:00-4:15 Professor Warren E. Adams, Clemson University
"A Hierarchy of LP Relaxations for 0-1 Polynomial Programming"

4:15-4:30 Dr. Marie-Christine Costa, Ministere de l'Education Nationale, Inst. d'Informatique d'Entreprise
"A Decomposition Method for the 3-SAT Problem"

4:30-4:45 Dr. Brenda Dietrich, IBM, Thomas J. Watson Research Center
"Extended Coefficient Reduction in 0-1 Programming"

4:45-5:00 Professor John N. Hooker, Carnegie-Mellon University
"Integer Programming From a Logical Point of View"

5:00-5:20 Coffee Break

5:20-5:35 Professor Brigitte Jaumard, Ecole Polytechnique Montreal
"Computational Results in Nonlinear 0-1 Programming"

5:35-5:50 Professor Martine Labbé, Erasmus University
"Voronoi Partition of a Network"

5:50-6:05 Professor Donna C. Llewellyn, Georgia Institute of Technology
"A Primal-Dual Algorithm for Integer Programming"

6:30- *Party at the home of Fred and Helen Roberts*

THE STATE UNIVERSITY OF NEW JERSEY
RUTGERS
RUTCOR · Rutgers Center for Operations Research
Hill Center for the Mathematical Sciences
New Brunswick · New Jersey 08903

ARIDAM IV

FOURTH ADVANCED RESEARCH INSTITUTE ON DISCRETE APPLIED MATHEMATICS

RUTCOR, Hill Center, Room 705

Program for Friday, June 2, 1989

9:00-10:00 Breakfast

Morning Session

Chairman: Professor Jeffry Kahn
RUTCOR and Department of Mathematics, Rutgers University

10:00-12:20 Seventh and eighth lectures by Professor Richard M. Karp
University of California at Berkeley
"Combinatorial Aspects of Parallel Computation"

12:20-4:00 Period reserved for lunch and free discussions among the participants

Afternoon Session

Chairman: Professor Dr. Gottfried Tinhofer, Technische Universität München

4:00-4:15 Professor Craig W. Rasmussen, University of Colorado at Denver
"Interval Competition Graphs"

4:15-4:30 Professor Jacques Carlier, Université de Technologie de Compiègne
"Timed Petri Net Schedules"

4:30-4:45 Professor Philippe Chrétienne, ONERA, Paris
"Scheduling Problems Over Distributed Architectures"

4:45-5:00 Professor Giulia Galbiati, University of Pavia
"Random Pseudo-Polynomial Algorithms and Problems"

5:00-5:20 Coffee Break

5:20-5:35 Professor Felix Lazebnik, University of Delaware
"On the Number of Irregular Assignments on a Graph"

5:35-5:50 Dr. Federico Malucelli, University of Pisa
"Quadratic Assignment Problem: an Enumerative Approach Based on the Ranking of the Solutions"

5:50-6:05 Professor Uri N. Peled, University of Illinois at Chicago
"Poset Matching: a Distributive Analog of Independent Matching"

6:05-6:20 Dr. Edward Sewell, Cornell University
"On Rank Facets for the Stable Set Polytopes"

THE STATE UNIVERSITY OF NEW JERSEY
RUTGERS

RUTCOR · Rutgers Center for Operations Research
Hill Center for the Mathematical Sciences
New Brunswick · New Jersey 08903

ARIDAM IV

FOURTH ADVANCED RESEARCH INSTITUTE ON DISCRETE APPLIED MATHEMATICS

RUTCOR, Hill Center, Room 705

Program for Monday, June 5, 1989

9:00-10:00 Breakfast

Morning Session

Chairman: Professor **Pierre Hansen**, RUTCOR, Rutgers University

10:00-11:00 First lecture by Professor **Alan J. Hoffman**
IBM Thomas J. Watson Research Center
in the series "Eigenvalue Bounds and Linear Programming"

11:00-11:20 Coffee Break

11:20-12:20 First lecture by Professor **Alan J. Hoffman**
in the series "Greedy Algorithms and Linear Programming"

12:20-4:00 Period reserved for lunch and free discussions among the participants

Afternoon Session

Chairman: Professor **Albert C. Williams**, RUTCOR, Rutgers University

4:00-4:15 Professor **Ann C. Mugavero**, St. John's University
"Graphs for Musical Analysis"

4:15-4:30 Professor **Chinh T. Hoang**
RUTCOR and Dept. Computer Science, Rutgers University
"An O(log n) Parallel Algorithm for Testing Bipartite Graphs"

4:30-4:45 Dr. **Laureano F. Escudero**, IBM Thomas J. Watson Research Center
"Optimal Workload Allocation for Parallel Unrelated Machines with Setups"

4:45-5:00 Professor **Yves Crama**, University of Limburg, Maastricht
"Maximum Intersection of Ellipsoids"

5:00-5:20 Coffee Break

5:20-5:35 Professor Dr. **Gottfried Tinhofer**, Technische Universität München
"Recent News Concerning Graceful Numbering of Trees"

5:35-5:50 Professor **Celso C. Ribeiro**, Catholic University of Rio de Janeiro
"The Minimum Rectilinear Steiner Tree Problem: Heuristics and a Computational Study"

5:50-6:05 Professor **M. Resa Emamy**, University of Puerto-Rico
"Isomorphism Problem for Some Subgraphs of the d-cube and Optimization of Certain Pseudo-Boolean Functions"

6:05-6:20 Dr. **Liping Sun**, Dept. Computer Science, Rutgers University
"Two Classes of Perfect Graphs"

6:20- *Wine and cheese party*

THE STATE UNIVERSITY OF NEW JERSEY
RUTGERS
RUTCOR · Rutgers Center for Operations Research
Hill Center for the Mathematical Sciences
New Brunswick · New Jersey 08903

ARIDAM IV

FOURTH ADVANCED RESEARCH INSTITUTE ON DISCRETE APPLIED MATHEMATICS

RUTCOR, Hill Center, Room 705

Program for Wednesday, June 7, 1989

9:00-10:00 Breakfast

Morning Session

Chairman: Professor Bahman Kalantari
RUTCOR and Dept. of Computer Science, Rutgers University

10:00-11:00 Professor Gyula O.H. Katona, Case Western Reserve University and Mathematical Institute of the Hungarian Academy of Sciences
"Convex Hulls of Some Classes of Hypergraphs"

11:00-11:20 Coffee Break

11:20-12:20 Professor Dorit S. Hochbaum, University of California-Berkeley
"Complexity of Nonlinear Optimization"

12:20-4:00 Period reserved for lunch and free discussions among the participants

Afternoon Session

Chairman: Professor Michael H. Rothkopf, RUTCOR

4:00-4:15 Professor Maurice Queyranne, University of British Columbia
"Scheduling Polyhedra"

4:15-4:30 Dr. Guoli Ding, RUTCOR
"2-Monotone Matriods, 3-Monotone Matroids and Threshold Matroids"

4:30-4:45 Professor Janny Leung, Yale University
"A Heuristic for Facility Layout"

4:45-5:00 Dr. Francois Margot, Ecole Polytechnique Fédérale de Lausanne
"Poset Scheduling Problems"

5:00-5:20 Coffee Break

5:20-5:35 Professor Kenneth P. Bogart, Dartmouth University
"Representations of Semi-Orders and Interval Orders"

5:35-5:50 Professor Charles R. Johnson, College of William and Mary
"Certain Partial Orders on Collections of Sets"

5:50-6:05 Professor Christian Ebenegger, Université de Genève
"Aggregation of Partial Orders"

6:30- *Backyard Party at Professor Al Williams' House*

THE STATE UNIVERSITY OF NEW JERSEY
RUTGERS
RUTCOR · Rutgers Center for Operations Research
Hill Center for the Mathematical Sciences
New Brunswick · New Jersey 08903

ARIDAM IV

FOURTH ADVANCED RESEARCH INSTITUTE ON DISCRETE APPLIED MATHEMATICS

RUTCOR, Hill Center, Room 705

Program for Thursday, June 8, 1989

9:00-10:00 **Breakfast**

<u>Morning Session</u>

<u>Chairman:</u> Professor **Michael D. Grigoriadis**
RUTCOR and Dept. of Computer Science, Rutgers University

10:00-11:00 First lecture by Professor **Alan J. Hoffman**
IBM Thomas J. Watson Research Center and Stanford University
in the series "Greedy Algorithms and Linear Programming"

11:00-11:20 Coffee Break

11:20-12:20 Second lecture by Professor **Alan J. Hoffman**
IBM Thomas J. Watson Research Center and Stanford University
in the series "Greedy Algorithms and Linear Programming"

12:20-4:00 Period reserved for lunch and free discussions among the participants

<u>Afternoon Session</u>

<u>Chairman:</u> Professor **András Prékopa**, RUTCOR

4:00-4:15 Professor **András Sebö**, University of Bonn
"Uniquely Colorable Perfect Graphs and the Clique Rank"

4:15-4:30 Professor **Paolo Nobili**, Carnegie Mellon University
"Anti-join Composition Operation for Set-covering Polyhedra"

4:30-4:45 Dr. **Michel Troyon**, RUTCOR
"Struction versus Basic Algorithm"

4:45-5:00 Dr. **Glenn Hurlbert**, Dept. of Mathematics, Rutgers University
"On the Antipodal Layers Problem"

5:00-5:20 Coffee Break

5:20-5:35 Professor **Ivo G. Rosenberg**, Université de Montréal
"A Continuous Approach to the Optimization of Pseudo-Boolean Functions"

5:35-5:50 Dr. **Ron Holsman**, The Weizmann Institute
"On the Product of Sign Vectors and Unit Vectors"

5:50-6:05 Professor **Pierre Hansen**, RUTCOR
"Sharp Bounds on the Order, Size and Stability Number of Graphs"

THE STATE UNIVERSITY OF NEW JERSEY
RUTGERS
RUTCOR · Rutgers Center for Operations Research
Hill Center for the Mathematical Sciences
New Brunswick · New Jersey 08903

ARIDAM IV

FOURTH ADVANCED RESEARCH INSTITUTE ON DISCRETE APPLIED MATHEMATICS

RUTCOR, Hill Center, Room 705

Program for Friday, June 9, 1989

9:00-10:00 Breakfast

Morning Session

Chairman: Professor Peter L. Hammer, RUTCOR

10:00-11:00 Third lecture by Professor **Alan J. Hoffman**
IBM Thomas J. Watson Research Center and Stanford University
in the series "Greedy Algorithms and Linear Programming"

11:00-11:20 Coffee Break

11:20-12:20 Fourth lecture by Professor **Alan J. Hoffman**
IBM Thomas J. Watson Research Center and Stanford University
in the series "Greedy Algorithms and Linear Programming"

12:30-2:00 *ARIDAM's Third Anniversary and Farewell Party*

ARIDAM IV - List of Participants

Name	Address
J. Abello	Computer Science Department Texas A&M University
W. E. Adams	Dept. of Math Sciences Clemson University
S. Amarel	Department of Computer Science Rutgers University
R. D. Armstrong	Graduate School of Management Rutgers University
A. Bagchi	RUTCOR Rutgers University
A. Ben-Israel	RUTCOR Rutgers University
C. Berge	E.R. Combinatoire Centre de Mathematique Sociale
D. Bienstock	Discrete Mathematics Group Bellcore
A. Billionnet	Ministere de l'Education Nationale Inst. d'Informatique d'Entreprise
D. H. Blair	Dept. of Economics Rutgers University
K. P. Bogart	Dept. of Mathematics & Computer Science Dartmouth College
G. Bohus	Dept. of Mathematics Rutgers University
E. Boros	RUTCOR Rutgers University
C. Cable	Department of Mathematics Allegheny college
J. Carlier	Universite de Technologie
P-C. Chen	RUTCOR Rutgers University

Name	Address
M-S. Chern	RUTCOR Rutgers University
Chi Wang	RUTCOR Rutgers University
P. Chretienne	ONREA
S. Cosares	Bellcore Room PYA 2J307
M-C. Costa	Ministere de l'Education Nationale Inst. d'Informatique d'Entreprise
Y. Crama	Dept. of Quantitative Economics University of Limburg
B. Dietrich	I.B.M. Thomas J. Watson Research Center
G. Ding	RUTCOR Rutgers University
O. Egecioglu	Dept. of Computer Science Univ. of California Santa Barbara
M. R. Emamy-K.	Departmento de Matematicas Universidad de Puerto Rico
L. F. Escudero	IBM, J. Watson Research Center P.O.Box 218
M. Farber	AT&T Bell Laboratories HO-3L-308
T. A. Feo	Dept. of Mech. Eng., OR Group University of Texas at Austin
A. S. Fraenkel	Dept. of Applied Mathematics Weizmann Institute of Science
G. Galbiati	Dip. Di Matematica University Di Pavia
G. Gallo	Dipartimento di Informatica University of Pisa

Name	Address
M. D. Grigoriadis	Dept. of Computer Science Rutgers University
J. Gustedt	Technische Universitat Berlin Fachbereich Mathematik (MA 6-1)
R. Haas	Dept. of Mathematics North Carolina State University
G. Hahn	Dept. d'informatique et de recherche operatinelle, Universite de Montreal
L. Hall	E40-179, Operations Research MIT
M. D. Halsey	Dept. of Mathematical Sciences Worcester Polytechnic Institute
P. L. Hammer	RUTCOR Rutgers University
P. Hansen	RUTCOR Rutgers University
C. T. Hoang	Dept. of Computer Science Rutgers University
D. S. Hochbaum	School of Business Administration University of California-Berkeley
A. J. Hoffman	IBM Thomas Watson Research Center Mathematical Sciences Dept.
F. Hoffman	Department of Mathematics Florida Atlantic University
R. Holzman	Faculty of Mathemaics The Weizmann Institute
J. N. Hooker	Grad.School of Industrial Administration Carnegie-Mellon University
G. Isaak	RUTCOR Rutgers University
B. Jaumard	GERAD-HEC Ecole Polytechnique Montreal

Name	Address
C. R. Johnson	Dept. of Mathematics College of William and Mary
D. S. Johnson	Room 2C-355 AT&T Bell Laboratoris
J. Kahn	Dept. of Mathematics Rutgers University
R. M. Karp	Dept. of E.E. & CS University of California,Berkeley
G. O. H. Katona	Dept. of Mathematics Case Western Reserve University
L. L. Kelleher	Mathematics Dept. Massachusetts Maritime Academy
H. Kellerer	Institut fur Mathematik Technische Universitat Graz
J. Kim	RUTCOR Rutgers University
S. R. Kim	
B. Korte	Abteilung Operations Research Universitat Bonn
M. Labbe	Econometric Institute Erasmus Uiversity
F. Lazebnik	Dept. of Mathematical Sciences University of Delaware
J. Leung	School of Management MIT, E53-339
W. Li	RUTCOR Rutgers University
K. Lih	RUTCOR Rutgers University
D. C. Llewellyn	School of Ind. and Systems Engineering Georgia Institute of Technology

Name	Address
S. H. Lu	RUTCOR Rutgers University
X. Lu	Dept. of Mathematics Rutgers University
J. R. Lundgren	Department of Mathematics University of Colorado at Denver
F. Maffray	RUTCOR Rutgers University
N. V. R. Mahadev	Dept. of Mathematics Northeastern University
M. Majidi	Dept. of Computer Science Rutgers University
F. Malucelli	Dipartimento di Informatica University of Pisa
F. Margot	Departement de Mathematiques EPF Lausanne
T. J. Marlowe	Dept. of Computer Science Seton Hall University
J. B. Mazzola	Fuqua School of Business Duke University
R. McLean	Dept. of Economics Rutgers University
N. Megiddo	IBM Almaden Research Center 650 Harry Road
R. H. Mohring	Technische Universitat Berlin Fachbereich Mathematik (MA 6-1)
C. L. Monma	Bell Communications Research 435 South Street
A. C. Mugavero	St. John University
N. Nasr	Dept. of Industrial Engineeiring Rutgers University

Name	Address
N. Nisan	Computer Science Lab. MIT
P. Nobili	GSIA Carnegie Mellon University
P. M. Pardalos	Computer Science Department The Pennsylvania State University
D. Peeters	Department of Geography Universite Catholique de Louvain
U. N. Peled	Dept. of Mathematics University of Illinois-Chicago
M. Preissmann	Laboratoire ARTEMIS
A. Prekopa	RUTCOR Rutgers University
M. Queyranne	Fac. of Commerce & Business Adm. University of British Columbia
C. Rasmussen	Dept. of Mathematics University of Colorado-Denver
A. Raychaudhuri	Dept. of Mathematics College of Staten Island, CUNY
C. C. Ribeiro	Dept. of Electrical Engineering Catholic University of Rio de Janeiro
F. S. Roberts	Dept. of Mathematics Rutgers University
S. Rosenbaum	Dept. of Mathematics Rutgers University
I. G. Rosenberg	Mathematiques et Statistique Universite de Montreal
M. H. Rothkopf	RUTCOR Rutgers University
C. Roucairol	INRIA Domaine de Voluceau Rocquencourt

ARIDAM IV - List of Participants

Name	Address
H. Safer	Operations Research Center M.I.T.
D. Sakai	RUTCOR
M. J. Saltzman	College of Business & Public Admin. University of Arizona
E. Sewell	School of OR&IE Cornell University
D. Shanno	RUTCOR Rutgers University
H. Shum	School of OR&IE Cornell University
D. Sotteau	School of Computer Science McGill University
X. Sun	RUTCOR Rutgers University
E. Taillard	DMA EPFL
X. Tang	RUTCOR Rutgers University
F. Tardella	Centre de Recherches Mathematiques Universite de Montreal
B. Tesman	Dept. of Mathematics Rutgers University
P. Tetali	Courant Institute New York University
G. Tinhofer	Institut fur Mathematik Technische Universitat Munchen
W. T. Trotter	Dept. of Mathematics Arizona State University
M. Troyon	RUTCOR Rutgers University

Name	Address
A. Ulkucu	AT&T Bell Labs
D. Wagner	Fachbereich Mathematik (MA 6-1) Technische Universitat Berlin
A. C. Williams	RUTCOR Rutgers University
H. Wolkowicz	Dept. of Combinatorics & Optimization University of Waterloo
D. Wooster	Dept. of Computer Science Bob Jones University
E. Zemel	Graduate School of Management Northwestern University
M. Zheng	RUTCOR Rutgers University